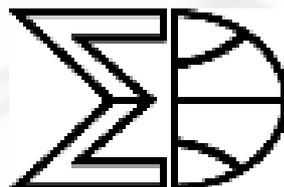


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MATHEMATICS COMPETITIONS



JOURNAL OF THE
WORLD FEDERATION OF NATIONAL
MATHEMATICS COMPETITIONS

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Articles (in English) are welcome.

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The aims of the Federation are:–

- 1. to promote excellence in, and research associated with, mathematics education through the use of school mathematics competitions;***
- 2. to promote meetings and conferences where persons interested in mathematics contests can exchange and develop ideas for use in their countries;***
- 3. to provide opportunities for the exchanging of information for mathematics education through published material, notably through the Journal of the Federation;***
- 4. to recognize through the WFNMC Awards system persons who have made notable contributions to mathematics education through mathematical challenge around the world;***
- 5. to organize assistance provided by countries with developed systems for competitions for countries attempting to develop competitions;***
- 6. to promote mathematics and to encourage young mathematicians.***

From the Editor

Welcome to *Mathematics Competitions* Vol 16, No 2.

Again, I would like to thank the Australian Mathematics Trust for its continued support, without which the journal could not be published, and in particular Heather Sommariva and Richard Bollard for their assistance in the preparation of the journal.

Submission of articles:

The journal *Mathematics Competitions* is interested in receiving articles dealing with mathematics competitions, not only at national and international level, but also at regional and primary school level. There are many readers in different countries interested in these different levels of competitions.

- The journal traditionally contains many different kinds of articles, including reports, analyses of competition problems and the presentation of interesting mathematics arising from competition problems. Potential authors are encouraged to submit articles of all kinds.
- To maintain and improve the quality of the journal and its usefulness to those involved in mathematics competitions, all articles are subject to review and comment by one or more competent referees. The precise criteria used will depend on the type of article, but can be summarised by saying that an article accepted must be correct and appropriate, the content accurate and interesting, and, where the focus is mathematical, the mathematics fresh and well presented. This editorial and refereeing process is designed to help improve those articles which deserve to be published.

At the outset, the most important thing is that if you have anything to contribute on any aspect of mathematics competitions at any level, local, regional or national, we would welcome your contribution.

Articles should be submitted in English, with a black and white photograph and a short profile of the author. Alternatively, the article can be submitted on an IBM PC compatible disk or a Macintosh disk. The preferred format is \LaTeX or \TeX , but any text file will be helpful.

Articles, and correspondence, can also be forwarded to the editor by mail to

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or by email to the address <warrena@amt.canberra.edu.au> or by fax to the Australian Mathematics Trust office, + 61 2 6201 5052, (02 6201 5052 within Australia).

Warren Atkins,
December 2003

* * *

From the President

Most of our attention currently will be directed to planning to attend the 2004 ICME-10 conference in Copenhagen.

At the WFNMC Conference in Melbourne we agreed to try to arrange to be more prominently on view than in past ICME conferences. In fact Conferences have been moved from a Topic Group status to that of Discussion Group. Whereas some of our members are unhappy that this will preclude the presentation of individual papers it is envisaged that it can open the discussion up more broadly, be well-structured and provide an opportunity for effective dissemination of ideas. I encourage those WFNMC members who can attend ICME to treat this discussion group as a good opportunity to interchange ideas and profile competitions in a positive light. I am cochairing this Discussion Group with Andre Deledicq and I invite you to make suggestions as to themes for the Group.

Regarding the WFNMC sessions so far we have two hours which will be needed for general business and discussion of our own. There is so far no opportunity for individual presentations here either. However, our Secretary Alexander Soifer may be able to obtain three extra hours for this purpose. The downside of this is that this time might be during or immediately after the happy hour.

In either case, if you wish to submit a paper it is worth doing so to Petar Kenderov, the Senior Vice President who also chairs the Program Committee. Petar's email address is

kenderovp@cc.bas.bg

I also suggest you monitor the WFNMC website at

<http://www.amt.edu.au/wfnmc.html>

where we will post information about the availability of such slots.

Finally I wish to remind you about ICMI Study 16 *Challenging Mathematics in and beyond the Classroom*. This is a rather broad Study which will include, among a number of other topics, an assessment

of competitions and their place in mathematics education. It is an opportunity for any member to make a submission in an attempt to contribute to the Study. The International Program Committee, of which Ed Barbeau (Toronto) and I are co-chairs, for this study, met in Modena, Italy, over 28 November to 01 December, and has made considerable planning progress. We are hoping to be able to release the Discussion Document in ICME-10 leading to a Study Conference to be held some time in 2006. Invitations to participate in the Conference will go out sometime in 2005.

Finally, it gives us great pleasure to announce the names of the winners of the Erdős Awards for 2004 in this edition. The winners are announced later in this edition. This award is very difficult to earn, with a quota of 3 awards every two years, and each time the selection committee has to sift through the cases of many nominees.

With best wishes

Peter Taylor
Canberra
December 2003

* * *

What is it about Mathematical Problem Solving that can be Taught?

Kaye Stacey



*Kaye Stacey is Foundation Professor of Mathematics Education at the University of Melbourne. Her doctorate from the University of Oxford is in number theory. She has written many practically-oriented books and articles for teachers as well as many research articles. Kaye's research interests centre on mathematical thinking and problem solving and the mathematics curriculum. Her first major book, *Thinking Mathematically*, published in 1982 with John Mason and Leone Burton, has been translated into French, Spanish, German and Chinese.*

Abstract

This presentation presents an overview of research on the aspects of mathematical problem solving that can be taught, with emphasis on teaching in the ordinary classroom setting. Mathematics competitions celebrate and encourage excellence in mathematical problem solving, and training for competitions often concentrates on imparting special techniques and recognizing features of 'competition-type' problems. In the context of an ordinary classroom with students of a range of mathematical abilities, the priorities and successful strategies are different. Programs need to be based around experience of tackling challenging problems in a supportive environment, with attention to general problem solving strategies and creating the motivation for students to reflect on the problem solving process, to build meta-cognitive control. We will discuss the characteristics of problems which are suitable for classroom

use, the goals of teaching problem solving and the nature of the success that can be expected in fostering mathematical thinking.

Broad concerns of research on problem solving

- 1975 Factors that make problems hard
- 1980 Comparison of expert and novice problem solving
- 1985 Metacognition, affect and beliefs
- 1990 Social influences and situated problem solving
- 2000 Fostering inquiry-based classrooms

(Schoenfeld, 1992 and others)

Well established differences between good and poor problem solvers

- Good problem solvers know more and know it differently
- Good problem solvers have well connected knowledge.
- Good problem solvers attend to structural features; poor problem solvers attend to surface features.
- Good problem solvers have better metacognition: understanding their strengths /weaknesses and better monitoring/control.

Recent curriculum history

- 1970 Polya and other writers
- 1980 Large curriculum movements: NCTM Agenda for the Eighties Mathematical Modelling and Applications
- 1990 Institutionalisation Example: Victorian Certificate of Education

- 2000 Integration into curriculum Mathematical reasoning in the USA ‘reform curriculum’

Ingredients for teaching problem solving

- Experience of tackling non-routine problems of some length
- Reflection on experience
- Learning basic heuristic strategies and good organisation
- ‘Can-do’ attitude and flexibility
- Teaching standard mathematics thoroughly—tension between strengthening intuitive strategies and teaching formal methods.

Tensions in Problem Solving in Classrooms

Challenge	\iff	Confidence
Using current maths	\iff	Using any maths
Doing problems (fun!)	\iff	Learning from doing (harder!)
Thinking and being stuck	\iff	Telling and teaching
Encouraging intuitive strategies	\iff	Learning formal, advanced methods

Good problems for class use

- Accessible
 - Must be able to start in a straightforward way
 - Good to be able to make progress with simple techniques
- Attractive
- Mathematically rich
 - Contains a surprise etc
 - Has plenty of deep structure
 - Good to illustrate important p.s. principles

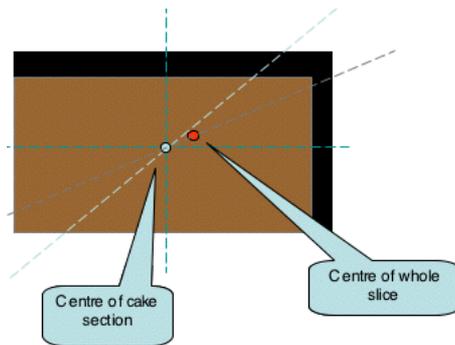
Dividing wedding cake among two

- Aim: Share icing and cake equally among two
- Rectangular slice, can't slice vertically, icing uniform but not necessarily same thickness top and side
- Constraint: Can only use one knife and one cut



Comments on Wedding Cake

- Strong tendency to horizontal/vertical bisection needs to be overcome for successful solution
- There is an easy 3-D solution: the challenge is to find the '2-D' solution



Problem solved by
joining these two points

Why is this not a good problem to use in teaching problem solving?

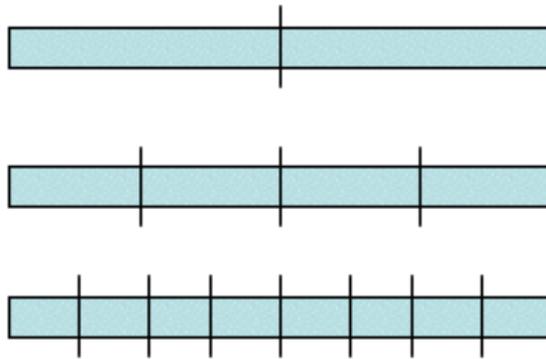
- Requires insight that ANY line through centre bisects area (not 'primary' knowledge)
- Initial psychological set towards vertical and horizontal lines. Solving requires determination to break the mental set—a useful lesson but hard to teach!
- Experimentation leads in the wrong direction

Paper Fold—a very accessible but rich problem for 10–14 year olds

Take a strip of paper and fold it in half, and then in half again and again.

Unfold.

How many creases would there be in the paper strip if you folded it 10 times?



See how hard this is to do directly

. more about this later.

STRIP	total creases	# folds	# thicknesses	# new creases
	0	0	1	
	1	1	2	1
	3	2	4	2
	7	3	8	4
	15	4	16	8
Now	many	patterns	emerge	easily

Many patterns emerge . . .

- # creases = # thicknesses - 1
- # creases = twice (previous #creases) + 1
- # thicknesses doubles as # folds increases
- # creases = previous #creases + #newcreases
- # creases = previous #thicknesses + #newcreases
- #creases = sum of all entries in #newcreases column
- Etc

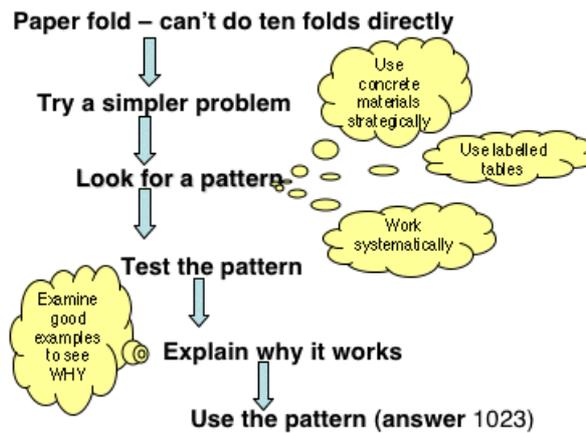
Need to look for reasons, not just spot the number patterns

STRIP	total creases	# folds	# thicknesses	new creases
	0	0	1	
	1	1	2	1
	3	2	4	2
	7	3	8	4
	15	4	16	8
	$2^n - 1$	n	2^n	2^{n-1}

How many creases with 10 folds?

- Theoretical answer: $2^{10} - 1 = 1023$
- Practical answer: Very hard to fold! Is the limit 7 folds?
- No, but is the limit 9 folds? Can mathematics help us find the limit?

- Don't forget the many simpler, but still important related problems: how fast does the thickness grow? How many halves would reach the moon? etc



Learning from Paper Fold

- Approached through 'Try a simpler problem' strategy
- Gathering data
 - Use the concrete materials purposefully
 - Mark systematically with coloured pencils etc
 - Record in labelled tables (so identifying variables)

- Record a variety of quantities, to help in pattern spotting
- Finding relationships
 - Seeing the general in the particular
 - Dont jump to conclusions (1,3,... odd numbers?)
 - Looking for reasons/structure, not just number patterns
 - Making and testing conjectures
- Power of mathematical modelling? Can we predict 9 folds?

Teachable strategies

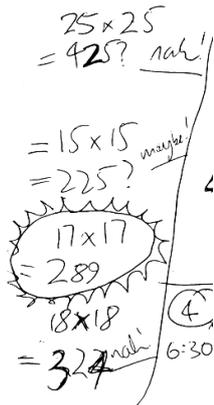
- Strategies for understanding a problem: finding out what I KNOW and what I WANT
 - Draw a diagram
 - Model/Act it out
 - Specialise–try a special case to see what happens
 - Restate the problem–various useful school strategies
- Strategies for working on a problem
 - Specialise–try a special case and look for the generality
 - Look for patterns (then explain them & use them)
 - Guess-check–improve
 - Be systematic. Use labelled tables.
 - Develop a good recording system

Study 1: Effects of problem solving teaching:

- Years 7 – 9, within normal school program, including non-routine problems and using everyday situations frequently.

- Differences were in favour of problem solving group:
 - Students worked more purposefully and kept more helpful written records.
 - Less prone to close quickly on an answer by combining numbers in a superficial way.
 - Used guess-check-improve strategy more effectively (and too often!)
 - No adverse effects on routine maths (depends on how much)

(Stacey, 1991, PME)



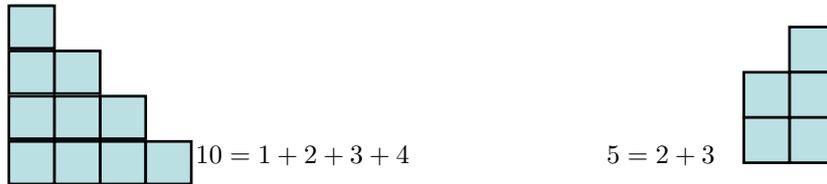
Which number between 1 and 150 when multiplied by itself produces the closest number to 300. (Problem set by teacher)

(Other problems showed real advantage with more organised guess and check)

Study 2: Staircase numbers

(Nick Scott, University of Melbourne, MEd)

- Some numbers, like 10 and 5, are sums of consecutive numbers. These are staircase numbers.



- Find which numbers are staircase numbers.
- Find a ‘recipe’ which explains how to write a number as a staircase number (in all possible ways).

Investigation of ‘Staircase Numbers’

- 10 solutions from graduate mathematics students, undergraduate Maths Olympian and pairs of mathematics undergraduates
- ‘Think aloud’ methodology
- About 20 – 30 minutes

1	0 + 1	Try Some Examples Some patterns quickly emerge	13	6+7
2			14	2+3+4+5
3	1+2		15	4+5+6
4			16	
5	2+3		17	8+9
6	1+2+3		18	5+6+7
7	3+4		19	9+10
8			20	
9	4+5		21	10+11
10	1+2+3+4		22	
11	5+6		23	11+12
12	3+4+5		24	7+8+9

1	0 +1	Extend Patterns	13	6+7
2 ?????	(-1)+ 0+1+2		14	2+3+4+5
3	1+2		15	4+5+6
4			16	
5	2+3		17	8+9
6 ??	0+1+2+3		18	3+4+5+6
7	3+4		19	9+10
8			20	
9	4+5		21	10+11
10	1+2+3+4		22	4+5+6+7
11	5+6		23	11+12
12	3+4+5		24	7+8+9

Main points of solutions

- All odd numbers are sums of two consecutive numbers, with informal demonstration (e.g. $21 = 10 + 11$, instructions for general number)
- Observing that powers of 2 are not staircase numbers
- Staircase numbers with an odd number of stairs are multiples of that number (and vice versa)
- Corresponding result for even factors (more complicated)
- Assembling all information to describe how to write a number as a staircase number from each of its factors. (Various depths of integration of solutions here)

Data from ten protocols

- XX Olympian undergraduate
- LR PhD student
- SF MSc student
- NW & JM pair of undergraduates
- Six other protocols, that follow these patterns
- Rough measure of success by giving % of outcomes (previous slide) which the interviewee established.

XX Olympian (80% of outcomes)

- Very quick
- Sought deep structure, operationalised with algebraic formulation
- Strategically important global assessments of progress
- Used targetted examples:
 - To reduce generality from algebraic expressions
 - Looking for the general in the particular
 - To identify algebraic errors

LR – PhD student – 30%

- Sought deep structure operationalised with algebraic formulation
- Used very few examples:
 - To become familiar with problem (briefly!)
 - To identify algebraic errors

- Examples were partially algebraic (e.g. $21 = m + (m - 1)$ where $m = (n + 1)/2, n = 21$)
- As a result, lost touch with the problem.
- Made early assessment that his approach was ‘not really getting anywhere’, but could not change.

$$\begin{aligned}n &= 3 \\ \text{What if } N \text{ is not div by } 3? \\ u &= 3 \quad (\text{ie } 4 \text{ cols}) \\ 4\left(m - \frac{3}{2}\right) &= N \\ \Rightarrow 2(2m - 3) &= N \\ 2m - 3 &= \frac{N}{2} \\ 2m &= \frac{N}{2} + 3 \\ m &= \frac{1}{2}\left(\frac{N}{2} + 3\right)\end{aligned}$$

LRs algebraic approach:

working on the case of
4 stairs

SF – MSc student – 80%

- Sought deep structure by collecting structural evidence from well-chosen examples.
- Made strategically important global assessments of progress.
- Used wide range of examples:
 - To look of patterns and suggest conjectures
 - As carriers of general structure
 - As a source of further directions for investigation.
 - Chose examples systematically and thoughtfully.
- Used algebra only as a notation

SF's table

- x starting stair, n number of stairs [$6 = 1 + 2 + 3$]...
- some corrections and errors

n	x	1	2	3	4
1		1	2	3	4
2		2 3	5	7 (=3+4)	9
3		5 6	9	12	15
4		9 10	14	18	22
5		14	21	25	30
6		22 21	29	31	39

NW & JM – undergrad pair – 40%

- Concentrated only on surface features: ‘what’ rather than ‘why’ or ‘how’.
- Used wide range of examples:
 - To collect data about what happens – they spotted many patterns
 - To explain a rule that they stated algebraically
 - Spotted many patterns, but did not attempt to prove anything.
- Comment when stuck: ‘at least we’re still collecting data’

All undergraduate pairs

- All protocols similar to NW & JM

- None more successful
- Shows strengths and weaknesses of school approach
 - Considerable benefits from looking at examples
 - Shows lack of focus on mathematical structure – not looking for the generality in the particular.

Characteristics of Protocols

	XX	LR	SF	NW&JM*
Category	Expert	Expert	Expert	Novice
Outcomes	80%	30% (knew less, better)	80%	40% (knew more)
Structure	Deep	Deep	Deep	Surface
Approach	Algebraic	Algebraic	Numeric	Numeric
Use of Examples	Few, targetted	Too few	Wide, targetted	Too many

* 5 pairs similar

Need a focus on mathematical structure:

- Orientation to asking ‘why’, not just ‘what’ (informal proving at all levels of schooling)
- Encouraging students to work with the ‘unexecuted expression’ (e.g. $1+2+3$, not 6) – this has links to the early algebra movement
- Rewarding/encouraging solutions that show structure (see next example)
- Big task for teacher education

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* * *

SEA-MO 2001: The Second SEA-MO Mathematics Olympiad

Krongthong, Khairiee, Tran Vui & Derek Holton



*Derek Holton is Professor of Pure Mathematics at the University of Otago and spends much of his time outside his teaching and research working with able mathematics students. He has written a number of books and articles for these students, including *Let's Solve Some Math Problems*. He has also led the New Zealand teams to the IMO half a dozen times.*

The South East Asian Mathematics Olympiad (SEA-MO) is another example of a regional Olympiad but it has a few twists on the regular format. Here we discuss the SEA-MO, give the questions from the second one and list the prize winners from both Olympiads.

Before we can start to say anything about SEA-MO, it's necessary to talk about SEAMEO and RECSAM. SEAMEO is the Southeast Asian Ministers of Education Organisation. Its ten member countries are Brunei Darussalam, Cambodia, Indonesia, Lao PDR, Malaysia, Myanmar, the Philippines, Singapore, Thailand and Vietnam. This organisation oversees various regional developments. One of these is RECSAM, the SEAMEO's Regional Centre for Education in Science and Mathematics which is housed in Penang. RECSAM involves itself in the in-service development of teachers at both primary and secondary school level. This is done through regular six-week long courses that relate to such things as pedagogy, computers and technology generally, remediation and enrichment. In addition it is able to offer customised courses for member countries either in that country or at RECSAM, or special services for countries outside the region. RECSAM is also involved in research and development in a number of areas relating to science, maths and technology at the school level.

As one of its initiatives in the area of enrichment at the secondary maths level, RECSAM organised the first SEA-MO in September 1998. All ten member countries of SEAMEO were involved. The top gold medal was awarded equally to Mr Somboon Siieawsharnpipat of Thailand and Mr Ngo Van Sang of Vietnam. The other gold medals that were presented went to Miss Trink Kim Chi of Vietnam and Mr Daniel Hue Yous of Singapore. The countries Vietnam, Singapore and Thailand were placed in that order on the teams' event.

The second SEA-MO was held in June 2001. The winners this year were Poh Wei Quan Julius, Singapore, Special Gold and Tan Wiewu Colin, Singapore, Thanasin Nampaisarn, Thailand and Pichamol Jirapinyo, Thailand, Gold. Singapore won the teams' event, Thailand was second and Indonesia was third.

Those of you familiar with the IMO will already have noticed a difference between IMOs and SEA-MOs in that the IMO is strictly an individual competition. There is no official team competition and certainly no team prizes. But the differences go further than this. A few of the other obvious differences are that countries send only four members rather than the six that go to an IMO. Then at a SEA-MO, there are three tests rather than the two of the IMO. Two of these are individual tests and one is a team test. In 2001, the first individual test was 90 minutes long and consisted of 20 short answer questions. Each of these questions was worth 1 point. The second individual test lasted for 180 minutes and is modelled on the IMO sets of questions that go for $4\frac{1}{2}$ hours. Here there were 5 problems each worth 7 points. Finally, after a longish break for lunch, the second day of competition concluded with the team test. Here there were 10 questions to be solved in 90 minutes. These questions were assigned 2 points each. Students work together in the team test and submit one set of answers per country.

The individual scores were taken from the first two tests while the team score was the sum of the scores of the team members in the individual tests plus four times the number of points from the team test.

Apart from those differences though, Leaders of countries at the IMO would find themselves very much at home at a SEA-MO. There is the usual sequestering of Team Leaders to produce the questions; there are

the usual co-ordination sessions; and there are the usual concerned faces waiting for scores to be posted.

We append here the questions from SEA-MO 98 and for SEA-MO 2001.

One of the things that the Jury of SEA-MO tries to do is to provide questions that will extend the more able students. This is so that there is a sufficient spread of marks at the top to separate out the prizewinners while at the same time challenging the best students and helping in their preparation of the IMO. On the other hand, we also try to produce questions that are accessible to all students. We hope to provide problems that all students can attempt successfully. Hence you will appreciate that the questions of SEA-MO 2001 were designed with a range of student ability in mind.

Short Answer Questions 1998

1. Find the sum of the following series:

$$\frac{2}{3} - 4 + \frac{4}{9} - \frac{4}{7} + \frac{8}{27} - \frac{4}{49} + \cdots$$

2. Find the unit digit of 7^{2020} .
3. A sphere is inscribed in a cube and another sphere is circumscribed about the same cube. What is the ratio of the volume of the smaller sphere to the larger sphere?
4. In a certain sequence of numbers, the first term is 1, and, for all n greater than or equal to 2, the product of the first n terms in the sequence is n^3 . Find the sum of the third and fifth terms in the sequence.
5. The function $f(x)$ is defined by

$$f(x) = \max\{4x + 1, x + 5, -2x + 2\}$$

for every real number x . Find the minimum value of $f(x)$.

6. Let $ABCD$ be a square. The arc of the circle with center D and radius DA cuts the diagonal BD at X . Similarly, the arc of the circle with center B and radius BC cuts BD at Y . If $XY = 2$, find the area bounded by the two arcs AXC and AYC .
7. In the equilateral triangle ABC , the points H, M, N, I are the midpoints of BC, AH, AC, NH respectively. If the area of the triangle IMH is 10, find the area of the triangle ABC .
8. Let $p(x)$ be a polynomial. The remainders of the division of $p(x)$ by x and $(x - 1)$ are 1 and 2 respectively. Find the remainder of the division of $p(x)$ by $x(x - 1)$.
9. The sides of a triangle are 7, 8 and 11 units. What is the area of the circle inscribed in the triangle?

10. Solve the equation

$$\sqrt{2x^2 - 4x + 9} - \sqrt{2x^2 - 4x - 12} = 3.$$

11. Let $A = \{p, q, r\}$ and $C = \{p, q, s, t\}$. Determine the number of sets B that can be constructed such that B is a subset of C and the intersection of A and B has exactly one element.
12. In how many ways can the letters in the word *PARALLEL* be arranged so that the order of the vowels A, A, E is unchanged? (All the letters must be used.)
13. Find the positive integral value of n for which $x^2 + x + 6$ equals n^2 for some positive integer x .
14. If α is a root of $x^{1999} = 1$ and $\alpha \neq 1$, find

$$1 + \alpha + \alpha^2 + \alpha^3 + \cdots + \alpha^{1998}.$$

15. Triangle ABC is right angled at A . The circle with center A and radius AB cuts BC and AC internally at D and E respectively. If $BD = 20$ and $DC = 16$, determine AC^2 .
16. Find the number of ordered pairs (x, y) , where x and y are positive integers, such that

$$\frac{xy}{x+y} = 1998.$$

17. Let A and B be the diagonally opposite vertices of a solid one-meter cube. What is the shortest distance from A to B along the surface of the cube?
18. Given $f(1) = 2$ and $2f(n+1) = 2f(n) + 1$, for $n = 1, 2, 3, \dots$, find n such that $f(n) = 1998$.
19. In an arithmetic sequence $t_1 = 98$ and $t_{13} = 89$.
Define $T_n = t_n + t_{n+1} + t_{n+2} + \cdots + t_{n+6}$.
Find the minimum value of $|T_n|$.

20. Find the value of

$$\sin^2 0^\circ + \sin^2 1^\circ + \sin^2 2^\circ + \cdots + \sin^2 90^\circ.$$

Individual Power Questions
(180 minutes)

1998

1. (a) Let $f(x)$ be a real valued function that is not defined at $x = 0$, such that $f(x) + 2f(1/x) = 3x$. Find all the solutions to the equation $f(x) = f(-x)$.
(b) The function $g(x)$ satisfies the equation $g(x) + g(y) = g(x + y) - xy - 1$, for every pair x, y of real numbers. If $g(1) = 1$, determine all integers n not equal to 1 for which $g(n) = n$.
2. If a, b, c are positive real numbers such that $a < b + c$, $b < a + c$, $c < a + b$ and $a + b + c = 2$, show that $a^2 + b^2 + c^2 + 2abc < 2$.
3. The date 14 July 1998 is a *special date* since if this date is written in the form 14/7/98, it may easily be seen that $14 \times 7 = 98$. Are there any other such dates in the year 1998? Explain.
How many special dates are there from 1 January 1900 to 31 December 1999?
4. Fourteen lines are drawn on the plane so that
 - (a) five of the lines all pass through a single point,
 - (b) three other lines are parallel to each other, and
 - (c) there are no other cases of three or more concurrent lines and no other cases of parallel lines.

Into how many regions do the lines divide the plane?

5. If n and k are positive integers and k is odd, show that $1^k + 2^k + 3^k + \dots + n^k$ is divisible by $1 + 2 + 3 + \dots + n$.

Each question is worth 7 points.

Team Test Questions
(60 Minutes)

1998

1. Let $\triangle ABC$ be an equilateral triangle with sides of unit length. Let $\triangle PQR$ be another equilateral triangle with sides of length a , $0 < a < 1$, inside $\triangle ABC$ such that $AB \parallel PQ$, $BC \parallel QR$ and $CA \parallel RP$. Let Q_1, R_1, P_1 be on the segments AB, BC, CA , respectively, such that AQ_1QP, BR_1RQ, CP_1PR are parallelograms. Determine the ratio

$$\frac{\text{area } AQ_1QP + \text{area } BR_1RQ + \text{area } CP_1PR}{\text{area } \triangle APP_1 + \text{area } \triangle BQQ_1 + \text{area } \triangle CRR_1}$$

in terms of a .

2. Find all ordered pairs of real numbers (x, y) which satisfy the following system of equations:

$$\begin{cases} x^3 + y^3 = 1 \\ x^4 + y^4 = 1 \end{cases}$$

3. Determine the minimum value of

$$f(x) = (3 \sin x - 4 \cos x - 10)(3 \sin x + 4 \cos x - 10).$$

4. Let P and Q be points outside of $\triangle ABC$ such that $\triangle BAP$ and $\triangle ACQ$ do not overlap $\triangle ABC$. Let R be a point inside $\triangle ABC$. Show that, if $\triangle BAP, \triangle ACQ$ and $\triangle BCR$ are similar isosceles triangles with bases AB, AC and BC , respectively, then the quadrilateral $AQRP$ is a parallelogram.
5. Find all positive integers a, b, c with $12 < a < b < c < 360$ such that:
- (a) the greatest common divisor of a, b, c is 12, and
 - (b) the least common multiple of a, b, c is 360.

6. For what values of the integer n is $11(14^n) + 1$ a prime?
7. Find a four-digit number which is a perfect square such that its first two digits are equal to each other and its last two digits are equal to each other.
8. The *integer part* $I(A)$ of a number A is the greatest integer which is not greater than A , while the *fractional part* $F(A)$ is defined as $A - I(A)$. Give an example of a positive number A such that

$$F(A) + F(1/A) = 1.$$

Could an example be found that is rational? Explain.

9. One of the numbers 1 or -1 is assigned to each vertex of a cube. To each face of the cube is assigned the integer which is the product of the four integers at the vertices of the face. Is it possible for the sum of the 14 assigned integers to be 0? Explain.
10. Three people $A, B,$ and C need to cross a bridge. A can cross the bridge in 10 minutes, B can cross in 5 minutes, and C can cross in 2 minutes. There is also a bicycle available which can carry one person at a time and any person can cross the bridge in 1 minute on the bicycle. Show that it is possible to get all of the people over the bridge in under 3 minutes.

Each question is worth 2 points.

Short Answer Questions
(90 minutes)

2001

1. Three consecutive terms of a geometric sequence are 3^{2x-1} , 9^x and 243. Find the value of x .
2. Two fair normal 6-face dice are thrown. Calculate the probability that the product of the scores is odd.
3. Students in a group dancing class are spaced evenly around a circle and are then counted consecutively from number 1. Student 20 is directly opposite student 53. How many students are there in the group?
4. The polynomial $f(x) = 2x^3 + Ax^2 + Bx - 3$ is exactly divisible by $(x - 1)$ and leaves a remainder 9 when divided by $(x + 2)$. Find the value of $(A - B)$.
5. A supermarket displays a certain soap by stacking the boxes in a ten-layered pyramid, each layer having a rectangular shape with one less box both in length and in width than the layer below. If the top layer consists of one row of six boxes, how many boxes are there in the total stack?
6. Let A, B, C, D be four concyclic points. AB extended and CD extended meet at P such that $PB = 4$, $PC = 5$, and $CD = 6.2$. Find AB .
7. The lengths of two corresponding medians of two similar triangles are 10 and 15, respectively. If the area of the larger triangle is 81, find the area of the smaller triangle.
8. Find the largest solution of $x^{2001} + 6x^{1997} = 7x^{1999}$.
9. For which real number a is the sum of the squares of the zeros of $x^2 - (a - 3)x - a - 2$ a minimum?

10. Ali, Balan and Chong participate in three different water sports.

- (a) If Ali sails, Balan surfs.
- (b) If Ali swims, Chong surfs,
- (c) If Balan does not swim, Chong sails.

What sport does Chong participate in?

11. Let

$$f(x) = (x - a)^2(b - c) + (x - b)^2(c - a) + (x - c)^2(a - b) \\ + (b - c)(c - a)(a - b),$$

where a , b , and c are real numbers. Find the numerical value of $f(2001)$.

12. If the sum of the cubes of three distinct numbers is three times their product, find the sum of the three numbers.

13. Given a cyclic quadrilateral in a circle of diameter x and with sides of length a , b , c , and x , find a cubic equation which x must satisfy.

14. Find the smallest positive integer n such that the product of 1999 and n has 2001 as its last four digits.

15. A 'nice' number is one which has two digits and is such that it is equal to the product of its digits plus the sum of its digits. How many 'nice' numbers are there?

16. Find the greatest number of regions the plane is divided into when n tangent lines are drawn to a circle.

17. Let $f_0(x) = \frac{1}{1-x}$ and $f_n(x) = f_0(f_{n-1}(x))$, for $n = 1, 2, 3, \dots$

Evaluate $f_{2002}(2001)$.

18. Given that
$$\begin{cases} U_{2n} = U_n \\ U_{2n-1} = 1 - U_n \end{cases}$$
 find U_{2001} .

19. Evaluate the product $\frac{7}{9} \times \frac{26}{28} \times \dots \times \frac{r^3 - 1}{r^3 + 1} \times \dots$

20. In the expansion of $(2 + x + \sqrt{x})^{2001}$, what is the sum of the coefficients of all integral powers of x ?

Each question is worth 1 point.

Individual Power Questions
(180 minutes)

2001

1. Prove that if n is a positive integer, then $(n^5 - n)$ is a multiple of 30.
2. $ABCD$ is a unit square. Let M be a point on AD and N on CD such that $\angle MBN = 45^\circ$.
Prove that $\sqrt{2} - 1 \leq S_{MBN} \leq \frac{1}{2}$, where S_{MBN} is the area of triangle MBN .
3. Let a be the integer consisting of m ones (i.e. $\underbrace{111 \dots 111}_m$) and let b be the integer $\underbrace{1000 \dots 000}_{m-1}5$, where there are $(m-1)$ zeros.
Show that $9(ab + 1)$ is a perfect square.
4. Let f be a function with the following properties:
 - (a) $f(n)$ is defined for every positive n ,
 - (b) $f(n)$ is an integer,
 - (c) $f(mn) = f(m)f(n)$ for all m and n ,
 - (d) $f(m) > f(n)$ for all $m > n > 0$,
 - (e) $f(2) = 2$.Find $f(2001)$.
5. (a) Let a_n be the largest integer of the form 3^t that divides

$$\frac{(3^2)!}{n!(3^2 - n)!} \quad \text{for } 0 \leq n \leq 3^2.$$

Show that

$$\sum_{n=0}^{3^2} \frac{1}{a_n} = \frac{10}{3}$$

(b) Let b_n be the largest integer of the form 3^t that divides

$$\frac{(3^k)!}{n!(3^k - n)!}, \quad \text{for } 0 \leq n \leq 3^k.$$

Calculate

$$\sum_{n=0}^{3^k} \frac{1}{b_n}$$

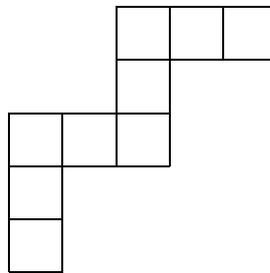
where n and k are integers.

Each question is worth 7 points.

Team Questions
(90 minutes)

2001

1. Put the numbers $1, 2, 3, 4, \dots, 9$ one per square, so that the sum of any three adjacent vertical squares and the sum of any three adjacent horizontal squares is 13.



2. Prove that the equation $x^2 + y^2 + 2xy - 2001x - 2001y - 2002 = 0$ has exactly 2001 solutions (x, y) for which x and y are both positive integers.
3. ABC is an equilateral triangle of side 1 unit and G is its centroid. Let M and N be variable points on AB and AC , respectively, such that the line segment MN passes through G . Find the minimum and maximum values of the area of the triangle AMN .
4. Let P be any point on a circle with centre O and radius r . For two points A and B on the circle such that AOB is the diameter, show that
- $PA \times PB$ is less than or equal to $2r^2$,
 - $PA + PB$ is less than or equal to $2r\sqrt{2}$.
5. Consider the arithmetic progression $1, 4, 7, \dots, 700$. When all the terms of this progression are multiplied together, what is the number of trailing zeros in the resulting product? (For example, the number 20 010 000 has four trailing zeros.)

6. Prove that

$$1 + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + \cdots + \frac{1}{\sqrt{2001}} > 87.$$

7. In triangle ABC , BM and CN are two medians that are perpendicular to each other. Prove that $b^2 + c^2 = 5a^2$ if $BC = a$, $CA = b$, and $AB = c$.

8. Let B be a set of 15 positive integers greater than 1 and less than or equal to 2001 such that no two integers in the set have a factor in common other than 1. Show that there is at least one prime number in the set B .

9. Find the number of solutions, in natural numbers x, y, z , and w , of the equation

$$x + y + z + w = 21, \text{ such that } y > w.$$

10. Let A be a set of rational numbers from the interval $(0, 1)$ that satisfy the following conditions:

(a) $\frac{1}{2} \in A$.

(b) If $x \in A$, then $\frac{x}{2} \in A$ and $\frac{1}{1+x} \in A$.

Prove that A consists of all rational numbers from the interval $(0, 1)$.

Each question is worth 2 points.

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Using Crossnumber Puzzles to Teach Computing Students

David Clark



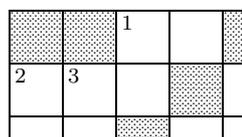
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Abstract: This paper describes how crossnumber puzzles can be adapted to provide a learning context for students in early programming courses. The adapted puzzles can be used initially to reinforce syntax, and later to give students opportunity to develop their ability to analyse code segments or functions

1 Crossnumber puzzles

Crossnumber puzzles have been around for nearly as long as their more popular cousins, the crossword puzzles. Whilst some crossnumber puzzles use uninteresting clues such as '1 across = 35×83 '. Others use interlocking clues which require inferences to be drawn and deductions to be made in their solution. Consider the following partial example with the single clue given. What deductions can be made from this? (Try it before reading on.)

2 across = 1 across + 1 down



It is not hard to see that 2 across must start with 1, and 1 across must finish with 0. Further, the first digit of 1 across must be at least 5, and the second digit of 2 across is even. But as 3 down cannot start with 0, it must start with 2, 4, 6 or 8 and the first digit of 1 across must be 6 or more.

There are some basic differences between crossnumber puzzles and crossword puzzles. The clues in crossnumber puzzles typically refer to more than one answer, and crossnumber puzzles are solved a cell at a time. A further difference is that there are usually only one or two entry points from which to start the puzzle. The example above contains a typical entry point - a 3-digit number which is the sum of two 2-digit numbers must start with 1. Another entry point is that a number which is a multiple of 5 must end with 0 or 5. If this last digit starts another number, 0 is ruled out.

Crossnumber puzzles are normally presented as 'brain teasers', requiring the ability to think logically, but requiring little or no mathematics. The only 'mathematics' involved is simple arithmetic; the operations $+$, $-$, $*$, $/$, and squares and cubes. A good example using interlocking clues but only simple arithmetic is Horsefield, [1].

2 A simple example

Here is a simple example which uses interlocking clues, but only the basic arithmetic operations of $+$, $-$ and $*$.

1	2			3	4
5			6		
	7	8			
9		10		11	
12				13	

Clues

$12a$ is twice $3a$
 $1a$ is the difference between $12a$ and $13a$
 $3d$ is 8 times $9d$
 $6a$ is the square of $1d$
 $6d$ is the sum of $7a$ and $8d$
 $4d$ is the sum of $1a$ and $12a$
 $10a$ is 5 times $5a$
 $2d$ is a multiple of $11d$
 $2d$ is the sum of $4d$ and $5a$

3 The University of Canberra Maths Day

Since 1978 the University of Canberra has run a maths day for final year secondary students from schools in the Canberra region. It is held in the University of Canberra gymnasium and is sponsored jointly by the University, its School of Mathematics and Statistics, and the Australian Mathematics Trust.

The UC Maths Day aims to be an exciting, enjoyable, light-hearted but nonetheless challenging day which provides an opportunity for mathematically able students to celebrate their talents. Up to forty teams, each of five students, compete in a number of events and there are trophies and prizes for the winning team overall, and for the teams with the greatest improvement over their school's performance in previous years.

There are four events on the day, each lasting 45 minutes. One the events is to solve a crossnumber puzzle. Each team is split into two, with one half seeing the across clues and the other half seeing the down clues. The crossword puzzle uses 9×9 grid, with about 60 cells. A solution guide is supplied which tells students which cell(s) can be deduced next. The crossnumber puzzles used in the UC Maths Day incorporate more interesting maths than simple arithmetic. For example arithmetic and geometric progressions, Fibonacci and triangular numbers, sequences, the Chinese remainder theorem, magic squares, integrals and Pythagorean triples have all been used in at least.

More details can be found in the Maths Day books [2]. The crossnumber puzzles used in the UC Maths Day were the genesis of the programming language based crossnumber puzzles described in this paper.

4 Clues in a computer language

The key idea in adapting a normal crossnumber puzzles to a programming language based crossnumber puzzle is simple. All clues are written as Boolean expressions which must evaluate to true. The table below shows the translation of the clues in the sample crossnumber puzzle above.

Clue in English	Clue in C / Java
$12a$ is twice $3a$	$12a == 2 * 3a$
$1a$ is the difference between $12a$ and $13a$	$1a == 12a - 13a$
$3d$ is 8 times $9d$	$3d == 8 * 9d$
$6a$ is the square of $1d$	$6a == 1d * 1d$
$6d$ is the sum of $7a$ and $8d$	$6d == 7a + 8d$
$4d$ is the sum of $1a$ and $12a$	$4d == 1a + 12a$
$10a$ is 5 times $5a$	$10a == 5 * 5a$
$2d$ is a multiple of $11d$	$2d \% 11d == 0$
$2d$ is the sum of $4d$ and $5a$	$2d == 4d + 5a$

5 A framework for learning

Whilst expressing clues in a computer language may provide some entertainment and some reinforcement of syntax, there is little pedagogical benefit. In order to be used effectively in teaching, we need to revisit Bloom's taxonomy of cognitive skills [3], namely knowledge, comprehension, application, analysis, synthesis and design. The latter three levels are sometimes referred to as 'deep learning'. When applied to learning programming, knowledge is knowing the syntax of language and the structure of a program, comprehension is the ability to trace data through code, application is the ability to write simple program or a component from a library, analysis is being able to understand what some code does, synthesis is the ability to use different components in an application, and evaluation is the ability to choose between alternate algorithms or designs.

In particular, the ability to analyse code is essential for a programmer. Maintenance of existing programs is one of the largest parts of the development life cycle, and this requires understanding of what a component does at a high level of abstraction. Only after a component is understood can the effect of changes be assessed. This ability can be taught very early. A typical early exercise is to ask ‘what is the output from this segment of code given this input?’. This is a comprehension question, and appropriate for learners. But the question can be turned into analysis by asking ‘what does this code do?’. If the code segment is fairly simple, such a question is accessible to quite new students.

Having questions appropriate to the level of students’ capabilities is necessary, and may well be sufficient for assessment. But it is not sufficient for learning. The message from problem based learning / situated cognition is that learning is more effective when it is placed in a context. The main point made in this paper is that crossnumber puzzles can provide a context for students to learn to analyse code.

6 Crossnumber puzzles providing a context for learning

This is achieved by extending the clues from simple Boolean expression to ones including integer or Boolean functions. Before a clue containing a function can be used to solve the puzzle, the function has to be analysed. Here are a couple of examples of functions which are capable of being analysed by students in their first programming subject.

```
int function1(int a, int b) {
    int result = 0;
    for (; a > 0; a--)
        result += b;
    return result;
} // function1

int function2(int a, int b) {
    while (a >= b) a -= b;
    return a;
} // function2
```

The appendix contains an example of a language based crossnumber puzzle with clues containing functions. Experience with constructing puzzles for the UC Maths Day has supplied a pool of simple mathematical operations suitable as expressing as functions.

7 Use in a programming course

Crossnumber puzzles have been used for the last couple of years in the subject 'Introduction to Software Engineering' at the University of Canberra. The students are in a Software or Computer Engineering Degree, which has a relatively high entrance requirement. The subject is highly algorithmic, and the type of analysis required to solve the puzzles fits well with the aims of the subject.

However, not all students regard crossnumber puzzles with unmitigated delight. This is addressed in two ways. Firstly the puzzles are optional. They are regarded as 'bonus' questions which give more marks to the students total and may make the difference in climbing over a grade boundary provided other essential criteria have been met. Secondly a solution guide is given to the students, similar to the guides used in the UC Maths Day.

The experience has been positive. About half of the students attempt the puzzles, and they almost submit completely correct solutions. The puzzles are given out a few weeks before the mid-semester test on which there are several 'what does this function do?' questions. These questions are answered quite well in the test.

8 Constructing language based crossnumber puzzles

It is hard to give advice on constructing crossnumber puzzles, either standard puzzles or with functions in the clues. I like to start with a list of functions that I will use. Sometimes this will require a particular feature in the pattern of the puzzle, such as a 3×3 square with the centre cell unused. (This feature has been used for including a magic square in a UC Maths Day puzzle.) More often there are no constraints on the pattern. I prefer to make up a pattern with mainly 2 and 3 digit answers, although I have used from 1 digit to 6 digit answers.

Once I have settled on a pattern, I decide where I want to put special features and / or answers to clues containing functions. Remember that clues interlock and so this may include more than one answer. Then I make one or two entry points and build the puzzle from there, incorporating my 'specials' as I come to them. I do this a cell at a time, adding interlocking clues to complete the answer later in the construction. As I go I make a note of the order in which I constructed the puzzle. This gives me the solution guide.

My main advice is to take care. A single error can cause the whole puzzle to unravel. When this happens any part of the puzzle that you salvage is a bonus. Another near requirement is someone independent who will test them out for you. I have a friend who is happy to solve them, but I have never been able to persuade him to check the solution guide. Fortunately once the puzzle is partly solved, the guide becomes less critical as several avenues open for making progress.

Once you have constructed a few, disasters are rare, but constructing them does require practise. This can be fun, as well as frustrating, but if that doesn't appeal there is a book of them available at puzzle.bandicoot.com.au and also at www.added.com.au/shop/Javaquiz.

9 References

- [1] Horsefield, L.G., *Cross Figure Puzzles Tandem*, London, 1972.
- [2] Brooks, M.S. (Editor), *University of Canberra Maths Day books for the years 1985 to 1991* are available from the Australian Mathematic Trust, PO box 1, Belconnen ACT 2616.
- [3] Bloom, B.S. (Ed.) (1956) *Taxonomy of Educational Objectives: Handbook 1, Cognitive Domain*. New York, Longman.

10 Appendix : Sample puzzle using functions

1	2		3	4	5
6		7		8	
	9		10		
11		12		13	
14	15		16		17
18				19	

Clues

1. $3 * 9a == 5 * 8a$
2. $18a == f08(10d)$
3. $13d == 2d + 11d$
4. $f09(19a, 13)$ 5. $f09(17d, 7)$
6. $15d == f01(17d, 19a)$
7. $6a \% 10 == 19a \% 10$
8. $7d == 1d * 1d + 100$ 9. $f09(4d, 1d)$
10. $7d == f01(6a, 1a)$
11. $12a / 50 == 12a \% 10$
12. $12a == 3a + 14a$
13. $5d == 16a + 3a$

Functions

<pre>int f08(int a) { int i, result = 0; for (i = 1; i ≤ a; i += 2) result++; return result; } // f08</pre>	<pre>int f01(int a, int b) { for (; a ≤ 0; a-) b++; return b; } // f01</pre>
<pre>boolean f09(int a, int b) { while (a ≤ b) a -= b; return (a == 0); } // f09</pre>	

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**The 44th International Mathematical Olympiad,
7-19 July 2003, Tokyo, Japan.**

It is difficult to forget the charming politeness of the people of Japan as well as their attention to beauty as manifest in the careful manicuring of their pine trees and in the presentation of their meals.

This was the setting of the 44th IMO. With 82 countries and 457 students, the IMO continues as a solid international event. Two new countries, Mozambique and Saudi Arabia, sent observers to this year's IMO. They intend to send teams in 2004.

Each invited country is permitted to send a team of up to six young amateur mathematicians who must be under 20 years of age and not enrolled in tertiary education. The team is accompanied by a Leader and a Deputy Leader. The Leader is involved in setting the paper, the Deputy in caring for the students up to the competition. Afterwards the Leader and Deputy combine to mark their students' papers and agree on scores in conjunction with local mathematicians.

The Team Leaders arrived a few days before the students and Deputies so as to put together a competition paper that would be both challenging and appealing. The Problem Selection Committee had already worked hard to reduce approximately one hundred proposed problems from all around the world to a short list of twenty-seven. In a way, the Team Leaders can get into the same spirit as the students at the IMO at this point, because they were given the short listed problems for about one and a half days to work on without solutions. Even easy problems can seem difficult when one is working under these conditions. Eventually solutions to all the problems were released to the Team Leaders. After this the Jury (all the Team Leaders as a committee) gradually selected the problems that would be on the paper. They were certainly mixed in subject and in difficulty, there being two each from geometry and number theory, and one each from algebra and combinatorics. Their origins were from Ireland, Brazil, Finland, Poland, Bulgaria, and France. Following this, translations were made into all the languages required by the students.

It is essential that confidentiality is maintained as regards the paper.

Hence, Team Leaders are kept separated from the rest of their teams until after the students have sat the competition. The main venue for the IMO was the National Olympics Memorial Youth Centre, Shinjuku, Tokyo. Once the students arrived there, the Leaders were moved to another location in Makuhari some two hours away to complete their deliberations.

There is however, one occasion where Teams get to see their Leaders prior to the exams, and that is during the opening ceremony. But “see” is all they get to do. The Team Leaders were paraded on stage, then there were speeches by those who had worked hard to make the 44th IMO happen. Finally the teams were all paraded on stage. It was very nice to see the teams from Cuba and Paraguay particularly applauded since they only sent a single contestant each.

The next two days were exam days. Students are allowed to ask questions in written form during the first half hour of the exam. Most have to do with notation and definitions, or even the student wanting to be extra careful that he has correctly understood the problem. One student, perhaps looking for some controversy asked “Can I use the axiom of choice” to which the response was “Yes.” Perhaps he had been waiting years to ask this question at the IMO. I hope it helped him solve the problem!

The exam turned out to be rather difficult this year. I am pleased to say that the coordination was very robust and did not soften to try and compensate for the difficulty of the exam. Actually this year the Team Leaders had the luxury of about a day to debate and provide input on the marking schemes from the Problem Selection Committee.

In the final analysis 210 students (46%) were awarded a bronze, 106 students (23.2%) were awarded a silver and 37 students (8.1%) were awarded a gold. A further 116 students received an honourable mention for gaining full marks on at least one question. Three students, namely one from China and two from Vietnam, achieved the outstanding result of a perfect score.

It was a privilege to have Shinno Naruhito, His Imperial Highness the Crown Prince of Japan present at the closing ceremony and who spoke to the audience along with Government Ministers during the prize

giving ceremony. Also memorable was the final perhaps unscripted act of inviting all the guides on stage and applauding them for looking after their teams so well. This finally seemed to turn more into a game show as a camera man and reporter started going around interviewing guides and contestants at random. There was one more award made at the banquet after the closing ceremony - that of the "Golden Microphone" awarded to the Team Leader who spoke the most during the Jury Meetings. This provides incentive for Team Leaders to speak less and more concisely at Jury meetings and thus avoid this prize!

The 2003 IMO questions and distribution of awards by country are shown below.

First Day

English version

1. Let A be a subset of the set $S = \{1, 2, \dots, 1000000\}$ containing exactly 101 elements. Prove that there exist numbers t_1, t_2, \dots, t_{100} in S such that the sets

$$A_j = \{x + t_j \mid x \in A\} \quad \text{for } j = 1, 2, \dots, 100$$

are pairwise disjoint.

2. Determine all pairs of positive integers (a, b) such that:

$$\frac{a^2}{2ab^2 - b^3 + 1}$$

is a positive integer.

3. A convex hexagon is given in which any two opposite sides have the following property: the distance between their midpoints is $\sqrt{3}/2$ times the sum of their lengths. Prove that all the angles of the hexagon are equal.

(A convex hexagon $ABCDEF$ has three pairs of opposite sides: AB and DE , BC and EF , CD and FA .)

Time allowed: 4 hours 30 minutes
7 points for each question

Second Day

English version

4. Let $ABCD$ be a cyclic quadrilateral. Let P , Q and R be the feet of the perpendiculars from D to the lines BC , CA and AB respectively. Show that $PQ = QR$ if and only if the bisectors of $\angle ABC$ and $\angle ADC$ meet on AC .
5. Let n be a positive integer and x_1, x_2, \dots, x_n be real numbers with $x_1 \leq x_2 \leq \dots \leq x_n$.

(a) Prove that

$$\left(\sum_{i=1}^n \sum_{j=1}^n |x_i - x_j| \right)^2 \leq \frac{2(n^2 - 1)}{3} \sum_{i=1}^n \sum_{j=1}^n (x_i - x_j)^2.$$

(b) Show that equality holds if and only if x_1, \dots, x_n is an arithmetic sequence.

6. Let p be a prime number. Prove that there exists a prime number q such that for every integer n , the number $n^p - p$ is not divisible by q .

Time allowed: 4 hours 30 minutes

7 points for each question

Distribution of Awards for the 2003 IMO

Country	Score	Gold	Silver	Bronze	Hon Men
Albania (4 members)	23	-	-	-	1
Argentina	91	1	1	2	1
Armenia	61	-	-	3	2
Australia	92	-	2	2	2
Austria	38	-	-	-	3
Azerbaijan	66	-	1	1	3
Belgium	70	-	1	1	3
Bosnia and Herzegovina	61	-	-	2	2
Belarus	111	1	2	2	-
Brazil	92	-	1	3	2
Bulgaria	227	6	-	-	-
Canada	119	2	-	3	-
China	211	5	1	-	-
Colombia	67	-	-	3	2
Croatia	80	-	-	3	3
Cuba (1 member)	14	-	-	1	-
Cyprus	23	-	-	-	1
Czech Republic	79	-	1	2	3
Denmark (5 members)	27	-	-	-	2
Ecuador	11	-	-	-	1
Spain	59	-	-	1	4
Estonia	47	-	-	-	3
Finland	43	-	-	1	2
France	95	-	2	2	1
Georgia	86	-	1	2	3
Germany	112	1	2	1	2
Greece	88	-	1	4	1
Hong Kong	91	-	2	2	1
Hungary	128	1	3	1	2
Iceland	33	-	-	1	1
Indonesia	70	-	-	2	4
India	115	-	4	1	1
Ireland	21	-	-	-	1
Iran	112	-	3	2	1
Israel (5 members)	103	-	2	3	-
Italy	50	-	-	1	4
Japan	131	1	3	2	-

Country	Score	Gold	Silver	Bronze	Hon Men
Kazakhstan	119	1	2	2	1
Kyrgystan	50	-	-	2	2
Korea	157	2	4	-	-
Kuwait (3 members)	8	-	-	-	-
Lithuania	49	-	-	2	2
Luxembourg (2 members)	25	-	-	1	1
Latvia	50	-	-	1	2
Macau	40	-	-	2	-
Malaysia (5 members)	26	-	-	-	1
Morocco	43	-	-	-	5
Mexico	64	-	-	3	1
Macedonia	54	-	-	2	3
Moldova	88	-	1	2	3
Mongolia	93	-	1	3	1
Netherlands	30	-	-	-	-
Norway	62	-	1	-	2
New Zealand	43	-	-	-	3
Paraguay (1 member)	0	-	-	-	-
Peru (4 members)	37	-	-	1	2
Philippines	9	-	-	-	-
Poland	102	1	2	-	2
Portugal	22	-	-	-	1
Puerto Rico (3 members)	23	-	-	1	-
Romania	143	1	4	1	-
Russia	167	3	2	1	-
South Africa	60	-	-	3	-
Serbia and Montenegro	101	-	3	1	2
Singapore	71	-	-	2	2
Slovenia	18	-	-	-	1
Sri Lanka (4 members)	4	-	-	-	-
Switzerland	26	-	-	-	1
Slovakia	77	-	-	4	2
Sweden	52	-	-	1	3
Thailand	111	1	1	3	1
Turkmenistan (4 members)	37	-	-	1	3
Trinidad and Tobago	33	-	-	-	2
Turkey	133	1	3	1	1
Taiwan	114	1	2	2	1

Country	Score	Gold	Silver	Bronze	Hon Men
Ukraine	118	1	2	3	-
United Kingdom	128	1	2	3	-
Uruguay (5 members)	29	-	-	-	2
United State of America	188	4	2	-	-
Uzbekistan	49	-	1	1	1
Venezuela (3 members)	10	-	-	-	-
Vietnam	172	2	3	1	-
Total (479 contestants)		37	69	104	116

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* * *

WFNMC International & National Awards

David Hilbert International Award

The David Hilbert International Award was established to recognise contributions of mathematicians who have played a significant role over a number of years in the development of mathematical challenges at the international level which have been a stimulus for mathematical learning.

Each recipient of the award is selected by the Executive and Advisory Committee of the World Federation of National Mathematics Competitions on the recommendations of the WFNMC Awards Sub-committee.

Past recipients have been: Arthur Engel (Germany), Edward Barbeau (Canada), Graham Pollard (Australia), Martin Gardner (USA), Murray Klamkin (Canada), Marcin Kuczma (Poland), Maria de Losada (Colombia), Peter O'Halloran (Australia) and Andy Liu (Canada).

Paul Erdős National Award

The Paul Erdős National Award was established to recognise contributions of mathematicians who have played a significant role over a number of years in the development of mathematical challenges at the national level and which have been a stimulus for the enrichment of mathematics learning.

Each recipient of the award is selected by the Executive and Advisory Committee of the World Federation of National Mathematics Competitions on the recommendations of the WFNMC Awards Sub-committee.

Past recipients have been: Luis Davidson (Cuba), Nikolay Konstantinov (Russia), John Webb (South Africa), Walter Mientka (USA), Ronald Dunkley (Canada), Peter Taylor (Australia), Sanjmyatav Urjintseren (Mongolia), Qiu Zonghu (China), Jordan Tabov (Bulgaria), George Berzsenyi (USA), Tony Gardiner (UK), Derek Holton (New Zealand), Wolfgang Engel (Germany), Agnis Andžāns (Latvia), Mark Saul (USA), Francisco Bellot Rosado (Spain), János Surányi (Hungary), Istvan

Reiman (Hungary), Bogoljub Marinkovich (Yugoslavia), Harold Reiter (USA) and Wen-Hsien Sun (Taiwan).

The general meeting of the WFNMC in Melbourne agreed, from 2003, to merge the above two awards into one award titled the Paul Erdős Award.

Requirements for Nominations for the Paul Erdős Award

The following documents and additional information must be written in English:

- A one or two page statement which includes the achievements of the nominee and a description of the contribution by the candidate which reflects the objectives of the WFNMC.
- Candidate's present home and business address and telephone/telefax number.

Nominating Authorities

The aspirant to the Awards may be proposed through the following authorities:

- The President of the World Federation of National Mathematics Competitions.
- Members of the World Federation of National Mathematics Competitions Executive Committee or Regional Representatives.

The Federation encourages the submission of such nominations from Directors or Presidents of Institutes and Organisations, from Chancellors or Presidents of Colleges and Universities, and others.

* * *

Multiple Choice Style Informatics

Jordan Tabov, Emil Kelevedzhiev & Borislav Lazarov



Jordan Tabov was an IMO participant and has been a team leader of the Bulgarian IMO team. He graduated from Moscow State University and since then he has been working at the Institute of Mathematics and Informatics of the Bulgarian Academy of Sciences. He is a Vice - President of the Tournament of the Towns and member of the editorial boards of several mathematics magazines and series of books related with mathematics competitions. In 1994 he was awarded the Paul Erdos Award of the WFNMC.

I. Introduction.

On 1st November 2001, the tenth edition of the Bulgarian mathematical tournament 'Chernorizets Hrabar' (briefly ChH) was held. As it was the first Bulgarian multiple choice competition and for a long period the unique math competition of this type, ChH has a number of specifics. This paper concerns a feature of ChH which probably distinguishes it from all multiple choice math competitions as far as the authors know and that is the presence of informatics problems.

During the last 20 years informatics has become one of the most developed branches of science closely related to mathematics. This is the main reason for beginning to separate student competitions in informatics. But another reason was the lack of any informatics related problems in mathematics competitions. The result is a complete separation of mathematics and informatics competitions not only in organization but even in student resources: as a rule students who attend math competitions avoid informatics competitions and vice versa. We thought that this was unacceptable 10 years ago and still have the same view now.

So ChH is an attempt to satisfy the demand for designing a math competition which includes enough informatics: at least to the extent of that in plane geometry, combinatorics and arithmetic. From this starting point we have the form of the competition almost uniquely determined – multiple choice problems competition.

But the question that arises is what kind of informatics is suitable for multiple choice problems. The only preliminary requirement for us was that an appropriate problem should allow a solution that can be found without using a computer. Since we started out in an unexplored area we had not any classification in mind. Now, in retrospect, we can say that we focus our attention on three basic directions, listed below.

II. Examples

(problem numbers refer to the Appendix 2).

1. Topics typical for computer programming (Informatics related questions).
 - (a) Given a commonly accepted description of an algorithm, compute its result for a given initial data – problems 93-14, 00-11.
 - (b) For a given algorithm, find out whether it will stop or will not, or compute how many times a given loop will run – problem 93-24.
 - (c) Find out a range of input data for which a given algorithm will complete correctly – problem 93-18.
2. Identification of an algorithm – what the algorithm should do? Guess the result, or describe the implemented method, recognizing in it some known algorithm – problems 93-12, 94-26, 96-27, 00-16, 00-17.

When a result of computation is asked, the difference between 1 and 2 is the following: In case 1, the result can be found by routine computations, while in case 2, one needs to recognize the action by means of applying some theoretical arguments.

3. Optimization – find out a minimal or maximal number of operations, which are necessary to compute some values, etc – problem 92-13.

III. Conclusions.

We have enough indicators to conclude that informatics problems (briefly IP) are relevant to our math competition. They enrich it in several directions.

1. The format of an IP differs in standards from the other math problems. Putting IP among usual-looking problems is a kind of shock.
2. The IP allows to give another view point to well-known math ideas. Sometimes this new point of view magically transforms Cinderella to a princess.
3. The algorithmic style of solving IP encourages students to apply it in pure math problems (see Appendix 2).

Statistics shows that IP are not a stumbling-block for the winners of ChH. As a rule the difficulty of an IP is of moderate level. Some are hard. Some are easy but they are classified by the jury as middle, because of their unusual look. As a direct result of their forms, all of the IP have a high discrimination factor.

It is hard to calculate the exact percentage of IP included in ChH. This is because some problems types are quite fuzzy to be strictly classified, e.g. problems concerning number systems. The proportion of IPs varies between 5% and 12%, but it is not so important how many IPs are contained in a ChH tournament test. Knowing there will be such problems, students (and teachers) include new areas in their training programs. And they like to do this.

Appendix 1. Features of the ChH tournament

Test of 30 multiple choice questions;

Penalty for each wrong answer;

Time allowed: 90 min;

No calculators or computers permitted;

Number of age levels: 3;

Over 2000 participants;

Where Held: about 10 in all main Bulgarian cities;

Frequency: yearly, every year on November 1.

Appendix 2. Selected problems and brief solutions.

92–13. What is the least possible number of multiplications by which a^{15} can be calculated for a given a (except for multiplications, any other arithmetic operations, such as raising to a power, are not allowed) ?

A) 3 B) 4 C) 5 D) 6 E) 14

Solution. a^{15} can be obtained from a after 5 multiplications in the following manner: First $a^2 = a \cdot a$, and then $a^3 = a \cdot a^2$. We have used two multiplications. The third is $a^5 = a^2 \cdot a^3$ and by 2 more multiplications we obtain a^{15} in the following way: $a^{10} = a^5 \cdot a^5$ and $a^{15} = a^5 \cdot a^{10}$. To prove that we cannot obtain a^{15} by 4 or less multiplications we make the full list of all possible cases

By 1 multiplication, only a^2 can be obtained.

By 2 multiplications: a^3 and a^4 .

By 3 multiplications: a^5 , a^6 and a^8 .

By 4 multiplications we can obtain a^7 , a^9 , a^{10} , a^{11} , a^{12} and a^{16} .

Answer. C).

93–12. The values of 128 and 52 are assigned to variables **a** and **b**, respectively. What will the variable **a** hold after a run of the following program part?

While **b**>0 do:

a:=**a**-**b** and if **a**<**b**, then exchange the values of **a** and **b**.

A) 1 B) 2 C) 4 D) 8 E) 0.

Solution. This part of the program implements Euclid's algorithm for finding out the greatest common divisor of two numbers (in the case, they are assigned to the variables **a** and **b**). When $a = 128$ and $b = 52$, the greatest common divisor is 4.

Answer. B).

93–14. An algebraic expression can be written in postfix notation (also called reverse polish notation) by putting first the operands and then their operation signs. For example, $a + b$ can be written as $ab+$, and $ab - c$ is an example of postfix notation of $(a - b) \cdot c$. What is the value of $abc + \cdot d \cdot$, where $a = 2, b = 3, c = 4, d = 5$?

A) 1 B) 7 C) 15 D) 68 E) 70.

Solution. We have $abc + \cdot d \cdot = a(7) \cdot d \cdot = (14)d \cdot = 70$.

Answer. E).

93–24. Variable **a** contains a real number in a floating point form. We assume its initial value is equal to 1. What will happen during the execution of the following loop?

While $\mathbf{a}+1 \neq 1$ do: **a**:=**a**/2.

- A) The execution of the loop will be repeated forever.
- B) The execution of the loop will never begin.
- C) The loop will be executed exactly once.
- D) The loop will be executed finitely many times, but more than once.
- E) None of the above.

Solution. Greater number of times the loop is repeated, closer to the zero variable's value a becomes. Due to the peculiarities of the floating point arithmetic, if two numbers have a great difference in their exponents, they have their sum closer (and even equal) to the number with the largest exponent. In our case, after several runs of the loop, the equality $a+1=1$ becomes true in the floating point arithmetic, and the loop stops running at that moment.

Answer. D).

94–18. A computer program adds the integer values $1, 2, \dots, n$ consequently to an integer variable x . The numbers are stored in a binary form with 15 digits at most. The initial value of x is 0. What is the greatest integer n for which the program can perform a correct computation?

- A) 126 B) 255 C) 256 D) 1024 E) 32768.

Solution. The sum of the first n positive integers is $S(n) = n(n+1)/2$. The greatest number that can be stored when 15 bits are used, is $2^{15} - 1$. Therefore, we should find such a number n , so that $S(n) \leq 2^{15} - 1$ and $S(n+1) > 2^{15} - 1$, i.e. $n(n+1) < 2^{16} \leq (n+1)(n+2)$. By probing we discover that for $n = 2^8 - 1 = 255$ both the inequalities are satisfied.

Answer. B).

94–26. Let the variables a and c have the values x and y , respectively. Let the variable b have the value 1. What will contain b after a run of the following program part:

While $c > 0$ do:

if c is an odd number then $b:=b.a$;

c:= the integer part of **c/2**

and **a**:=**a.a**.

A) x^y **B)** x^{y-1} **C)** $x.y$ **D)** $x - y$ **E)** x^2 .

Solution. If $y = b_n \cdot 2^n + b_{n-1} \cdot 2^{n-1} + \dots + b_1 \cdot 2^1 + b_0 \cdot 2^0$, $b_k \in \{0; 1\}$, then $x^y = x^{b_n \cdot 2^n} \cdot \dots \cdot x^{b_1 \cdot 2^1} \cdot x^{b_0 \cdot 2^0}$. The algorithm is storing consequently in the variable **b** the results of the multiplications from right to left in this product, and this is doing only for those terms, for which $b_k = 1$. After each step, the values x, x^2, x^4, x^8, \dots appear in **a**. The value of **c** (after deleting the remainder) is divisible by 2, and the remainders themselves, when they are divided by 2, are the numbers b_0, b_1, \dots

Answer. A).

96–27. Variables **x** and **y** store numbers in a floating point form. What will be the value stored in the variable **x** after a run of the following program part:

```
x = 0; y = 9999;
```

```
While x ≠ y repeat
```

```
{
```

```
  x = y;
```

```
  y = square root of (9999+2*x);
```

```
}
```

A) 0 **B)** 1 **C)** 99 **D)** 101 **E)** 9999

Solution. The program performs a repeating process $x_{n+1} = \sqrt{9999 + 2x_n}$, $n = 0, 1, 2, 3, \dots$, $x_0 = 9999$. Obviously $x_n \geq 0$ for $n = 0, 1, 2, 3, \dots$. The process is convergent because of the condition

$$x_{n+1} - x_n = \sqrt{9999 + 2x_n} - \sqrt{9999 + 2x_{n-1}}$$

$$= \frac{2(x_n - x_{n-1})}{\sqrt{9999 + 2x_n} + \sqrt{9999 + 2x_{n-1}}}$$

Therefore $x_{n+1} - x_n < 0 \iff x_n - x_{n-1} < 0 \iff \dots \iff x_1 - x_0 < 0$.
 But $x_1 = \sqrt{3 \cdot 9999} < 9999 = x_0$. From there $x_1 - x_0 < 0$ and $x_{n+1} - x_n < 0$, i.e. $x_{n+1} < x_n \forall n \in \mathbf{N}$. Thus the sequence $\{x_n\}$ is decreasing, bounded from below and hence convergent.

The limit satisfies the equation $x^2 = 9999 + 2x$, whose positive root equals 101.

Answer. D).

2000–11. The disk subdirectory `ChernorizecHrabar` contains the following files `ch1.imm`, `ch2.imm`, ..., `ch2000.imm`, all sorted by name. How many consecutively placed files has Albena marked, if the first one is `ch5.imm` and the last one is `ch51.imm`:

- A) 3 B) 13 C) 46 D) 47 E) none of these

Solution The marked files are `ch5.imm`, `ch50.imm`, `ch500.imm`, `ch501.imm`, `ch502.imm`, ..., `ch509.imm`, `ch51.imm`.

Answer. B).

2000–16. How many numbers will be printed after the execution of the following program:

```
10 FOR X=-10 TO 10
20 IF X*X-9*X-22=20 THEN PRINT X
30 NEXT X
```

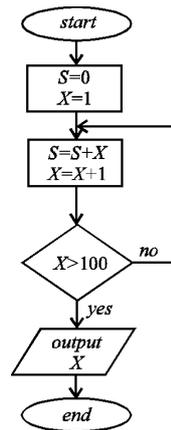
- A) 0 B) 1 C) 2 D) 3 E) 21

Solution The program is designed to print out the integer roots of the equation $x^2 - 9x - 42 = 0$ located in the interval $[-10; 10]$; but the equation has no such roots.

Answer. A).

2000–17. Which number will be printed out after the execution of the algorithm shown on the figure:

- A) 101
- B) 4096
- C) 5050
- D) 10100
- E) 1



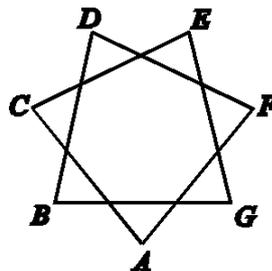
Solution The algorithm finds and prints out the sum of all the integers lying between 1 and 100. This sum is 5050.

Answer. C).

Appendix 3.

93–6. What is the sum of the angles A, B, C, D, E, F, G of the star $ABCDEFGG$ shown on the figure?

- A) 180°
- B) 270°
- C) 360°
- D) 540°
- E) 630°



Solution: Using a computer oriented style of thinking: Imagine you are a turtle, a character well known to the students learning Logo programming language. So, we are using so called ‘turtle graphics’. Visiting consequently every vertex along the path A, C, E, G, B, D, F , you are turning in an anti-clockwise direction, each time to the some

extent. The sum of all the measures of changing directions gives the answer to the problem.

Let us first consider a special case when the star is regular, i.e. it can be inscribed in a circle. So we have 7 turning points, each yields the same angle of $\alpha = (1/2) \cdot 360^\circ \cdot (3/7)$. The sum is equal to $7 \cdot \alpha = (3/2) \cdot 360^\circ = 540^\circ$.

In the general case, the exact measure in degrees at each turn is unknown, but at the end of the process, we can see that the turtle head directs to the same straight line as it was in the beginning. Hence, the accumulated measure is a multiple of 180° . Due to the reason of continuity between the “regular shape” of the star and its “general shape”, we can conclude there is no jump in measure while changing the shape, i.e. the multiple factor is the same as for the “regular shape”, hence the answer is 540° .

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* * *

Tournament of Towns Corner

Andrei Storozhev



Andrei Storozhev is a Research Officer at the Australian Mathematics Trust. He gained his PhD at Moscow State University specializing in combinatorial group theory. He is a member of the Australian Mathematics Competition Problems Committee, Australian Mathematics Olympiad Committee and one of the editors of the 'Mathematics Contests - The Australian Scene' journal.

Selected Problems from the First Round of Tournament 25

In the first round of Tournament 25, both Junior and Senior O Level papers consisted of five problems, while both Junior and Senior A Level papers were made up of seven problems. As usual, the problems were very challenging and required non-standard ideas in order to be solved. Here are selected questions with solutions from the first round of Tournament 25.

1. For any positive integer n , consider the greatest odd divisor of each of the numbers $n + 1$ to $2n$ inclusive. Prove that the sum of these n divisors is equal to n^2 .

Solution. For any positive integer k , define $o(k)$ to be the largest odd divisor of k . Note that $o(k) = o(\ell)$ implies that one of k and ℓ divides the other. Let n be any positive integer. No two of the n numbers from $n + 1$ to $2n$ divide each other. Hence these n odd numbers $o(n + 1), o(n + 2), \dots, o(2n)$ must be distinct. Since they are all less than $2n$, they must be $1, 3, \dots, 2n - 1$ in some order, so that their sum is n^2 .

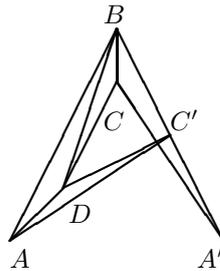
2. There is a counter on each of the leftmost 25 squares of a horizontal $1 \times N$ board. The counters are numbered from 1 to 25 from left to right. A counter may move to the right, but never to the left. It can either move to the next square if it is vacant, or jump over a counter occupying the next square to the square immediately beyond, provided that it is vacant. Determine the smallest value N such that all the counters can be moved so that they again occupy a continuous block of 25 squares, but numbered from 25 to 1 from left to right.

Solution. We first show that $N \leq 50$. Move counter 25 to space 26. Next, we move the other odd numbered counters in descending order. Each jumps as far as it can go and then make one final move. So counter 23 will be in space 28, counter 21 in space 30, and so on, until counter 1 goes to space 50. Then we move the even numbers counters in ascending order. Each makes an initial move and then jump as far as it can go. So counter 2 will be in space 49, counter 4 in space 47, and so on, until counter 24 goes to space 27. Now the counters occupy spaces 26 to 50 in reverse order. Clearly, we cannot have $N \leq 48$ as otherwise counter 25 have to move to the left. If $N = 49$, counter 25 does not move at all, counter 24 must occupy space 26, and nothing else can get to the right of them.

3. Each face of a tetrahedron is a piece of cardboard. Three of the edges are cut along so that each piece of cardboard lies flat. Could it happen that some pieces of cardboard overlap?

Solution. It is possible. Consider the points $A(0, 0)$, $B(40, 80)$, $C(40, 60)$, $D(20, 20)$, $A'(80, 0)$ and $C'(60, 40)$ in the coordinate plane. Note that C' lies on $A'B$. Since $AB = A'B = \sqrt{40^2 + 80^2}$, $AC' = A'C = \sqrt{60^2 + 40^2}$ and $CD = C'D = \sqrt{40^2 + 20^2}$, these six points are the vertices of a net for a tetrahedron $ABCD$ with base BCD . Cut along the edges AB and AC so that the face ABC flattens into $A'BC$. Then cut along CD so that the face BAD can lie flat, bringing the face ACD down as $AC'D$. As observed

earlier, $A'BC$ and $AC'D$ overlap.



4. An increasing arithmetic progression consists of one hundred positive integers. Is it possible that every two of them are relatively prime?

Solution. Let the arithmetic progression be $\{a_k\}$ where $a_k = k(101!) + 1$ for $k = 1, 2, \dots, 100$. Let d be the greatest common divisor of a_i and a_j where $i < j$. Then d divides $a_j - a_i = (j - i)(101!)$. If d has a prime divisor p , then $p \leq 101$. On the other hand, since p divides $i(101!) + 1$, we must have $p > 101$. This contradiction shows that d has no prime divisors. It follows that $d = 1$.

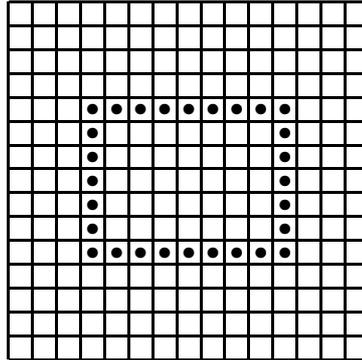
5. Several squares on a 15×15 chessboard are marked in such a way that a bishop placed on any square of the board attacks at least two of the marked squares. Find the minimal number of such marked squares. (A bishop attacks any square on the two diagonals that contain the square on which it stands, including that square itself.)

Solution. The answer is 28.

Consider bishops placed in the 56 squares along the edge of the board. Each must attack at least 2 of the marked squares. On the other hand, any square can be attacked by at most 4 of these bishops. Hence there must be at least $(56 \times 2) \div 4 = 28$ marked squares.

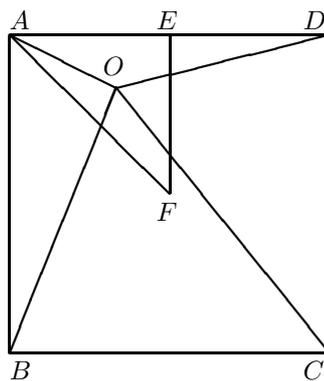
The diagram below shows that it is possible to mark 28 squares so that a bishop placed on any square of the board attacks at least

two of the marked squares.



6. A point O lies inside a square $ABCD$. Prove that the difference between $\angle OAB + \angle OBC + \angle OCD + \angle ODA$ and 180° does not exceed 45° .

Solution. Let E be the midpoint of AD and let F be the centre of $ABCD$. We may assume by symmetry that O is in triangle AEF . Hence $OA \leq OD$ and $OC \geq OB$, so that $\angle ODA \leq \angle OAD$ and $\angle OBC \geq \angle OCB$. Now $90^\circ = \angle OAB + \angle OAD \geq \angle OAB + \angle ODA \geq \angle OAB \geq 45^\circ$ while $90^\circ + 45^\circ \geq \angle OBC + \angle OCD \geq \angle OCB + \angle OCD = 90^\circ$. Addition yields the desired result.



World Wide Web

Information on the Tournament, how to enter it, and its rules are on the World Wide Web. Information on the Tournament can be obtained from the Australian Mathematics Trust web site at

<http://www.amt.canberra.edu.au>

Books on Tournament Problems

There are four books on problems of the Tournament available. Information on how to order these books may be found in the Trust's advertisement elsewhere in this journal, or directly via the Trust's web page.

Please note the Tournament's postal address in Moscow:

NN Konstantinov
PO Box 68
Moscow 121108
RUSSIA

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University of Canberra ACT 2601
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Winners of Erdős Award Announced

The WFNMC Executive has approved three Erdős Awards to be presented in 2004 as a result of an extensive investigation of nominations by the Awards Committee chaired by Ron Dunkley.

They are

Warren Atkins, Australia

Warren Atkins is one of four Australian mathematicians who together created what is today the Australian Mathematics Trust. In the myriad of activities generated through this organization, Warren has served in various capacities. Among them, he has been continuously a member of the Management Committee. He has been Chair of the Australian Mathematics Foundation. He has been an appointed representative to the Australian Mathematical Olympiad Committee. For many years he has been chairman of the Problems Committee of the Australian Mathematics Competition, where his leadership has led to interesting and challenging papers for students.

At the founding meeting of the Federation in 1984 he undertook the task of editing the Foundation Newsletter, and was named Editor of the Federation journal *Mathematics Competitions* which evolved from the newsletter, a role he has maintained to this day. As an author he has collaborated on numerous research articles dealing with student performance, and has authored a book on problem solving.

While his contributions have generally been behind the visible public screen, they have been significant, and his efforts have been a major factor in the creation and growth of the Australian Mathematics Competition.

André Deledicq, France

André Deledicq has established an enviable record in mathematics education. While he is known internationally for his work with the game-contest Kangourou, he has also made magnificent contributions in writing, publishing, teaching and lecturing.

In 1991 he created, in collaboration with Jean-Pierre Boudine, the contest Kangourou, with 120,000 participants. By 1993, when he was directing the operation himself, enrolment had passed 300,000, and by 1996, when other European and South American countries were included, enrolment passed one million annually. He has made Kangourou one of the largest and certainly one of the most innovative competitions in the world.

But this is not his main contribution. His major strength and interest is in popularizing mathematics at the school level, often through mathematical publications. To this end he has written and published, through a company he founded, a vast number of books, booklets, and posters that are cleverly written and appealing, and that have been distributed to hundreds of thousands of students.

As a teacher and lecturer he is enthusiastic, charming, and able to excite his audience. He has taught at the school level and in university. He has lectured in mathematics, computer science, and applications of technology in mathematics.

He has been involved in the training of teachers at the junior and senior high school level. Through his efforts mathematical competitions, rallies, clubs and other actions of widely diverse nature now flourish in France.

Patricia Fauring, Argentina

For more than sixteen years Patricia Fauring has been at the centre of mathematics competition activities in Argentina, working with students at all levels. Under her leadership and guidance Argentina has created and developed national and international events.

At the national level she is the central figure in a series of annual competitions involving more than one hundred thousand students each year. She has been the dominant figure in the development of the Ibero-American, South American and Southern Core nations competitions which are novel, innovative, and have had a significant impact on the development of mathematical problem-solving abilities among young Spanish and Portuguese-speaking students.

Among these are the Olympiad de Mayo or May Olympiad, held by correspondence for students aged thirteen to fifteen years, and the Olimpiada Rioplatense which brings together students from grade six to grade thirteen from the countries of the Rio de la Plata. Possibly the most innovative of her creations is the Frontier Tournament group of competitions, involving students in towns along the borders of Argentina and its neighbours, and designed to stimulate mathematical activity in outlying areas.

Patricia has been the principal mathematician involved in training Argentine teams for the IMO and other international events, where they have done respectably. She was also the organizer of the very successful 1997 IMO in Mar del Plata and has been elected to the IMO Executive Board.

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