A REPORT ON THE 29th IMO (9 - 21 JULY, 1988)

P.J. O'Halloran

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1. Introductory Comments

The 29th International Mathematical Olympiad was hosted in Australia from 9-21 July, 1988. It was the first time that an IMO was held not only in the southern hemisphere but also in the Asian/Pacific region. A record number of 268 contestants from 49 countries and territories plus representatives from another 9 countries and territories participated. For the first time there were more non-European teams (25) then European teams (24) participating at an IMO.

In all, 120 contestants from 38 of the 49 participating countries achieved medals (17 gold, 38 silver and 65 bronze ... see **Table Four**, 1988 IMO Distribution of Awards) with a special prize being awarded to a Bulgarian contestant for an outstanding solution to one of the most difficult questions ever set in an Olympiad (see **Table One**: 1988 IMO Score Frequency/Question and **Table Two**: Discrimination Factors of 1988 IMO Questions). The 1988 IMO's questions were considered to be more difficult than for the last few Olympiads (see **Table Five**: Comparison of the 1981 - 1988 IMO Results).

The six questions are given at the end of this article.

A new achievement category created at the Olympiad was an "Honourable Mention" award. This was given to contestants who obtained a perfect score (7 marks) for at least one IMO question and had not received a medal. A pleasing feature of this new award is that, as a result, another 7 countries were named as having team members who had received some recognition.

2. Some 29th IMO Statistics Tables

Another important new feature at the 1988 IMO was a 50-page book of 29th IMO statistics available within 3 hours of the Jury deciding the cut-off scores for the various medals. These statistics include information about previous IMO participation of contestants (approximately 20%), achievement by age (average scores of contestants' groups decrease with age), discrimination factors of questions, average age of medallists (gold medallists 17.3 years, silver 17.9 years and bronze 17.10 years) and so on.

A flavour of some of the statistics is reflected by the Tables given below. These statistics, the 90 question proposals (and their solutions) which were submitted to the Jury, are included in the 250-page book "An Olympiad Down Under - A Report on the 29th International Mathematical Olympiad in Australia". It costs $A20 (Australia and S. Pacific) and $A24 (elsewhere) and is available from the Australian Mathematics Competition, Canberra CAE, PO Box 1, Belconnen ACT Australia 2616.

TABLE ONE:

1988 IMO Score Frequency/Question (268 Candidates)

Marks	Number of Contestants per Mark per Question					
	Q1	Q2	Q3	Q4	Q5	Q6
0	27	92	75	136	76	189
1	56	16	132	25	35	57
2	19	13	13	12	23	3
3	24	19	1	17	10	5
4	22	25	3	3	20	1
5	14	24	4	3	11	1
6	8	7	9	6	7	1
7	98	72	31	66	86	11
Mean	3.9	3.2	1.7	2.3	3.3	0.6

Table One displays the spread of score frequencies obtained by the contestants for each question. The statistics clearly show the level of difficulty for each question. Of particular interest is the distribution of score frequencies for Question Six. This was one of the more difficult questions in the recent history of the Olympiads.

TABLE TWO:

Discrimination Factors of 1988 IMO Questions

Group of Contestants	Question						# in Group
	Q1	Q2	Q3	Q4	Q5	Q6	
Gold Medallists	0.00	0.00	0.00	0.00	0.00	0.75	17
Gold and Silver	0.06	0.25	0.69	0.44	0.19	0.63	55
All Medallists	0.34	0.47	0.59	0.66	0.31	0.38	120
All Contestants	0.91	0.84	0.54	0.76	0.82	0.19	268

Table Two gives the biserial discrimination index for each question. To obtain this index, the contestants are first ranked in overall ability. Then for a particular question, the number of marks greater than or equal to 4 for the top 25% of contestants is compared with the bottom 25% of contestants. The index can take values from -1 or +1. A positive value indicates that more contestants in the top group obtained 4 or more marks than contestants in the bottom group. It is considered that questions with discriminating factors of at least 0.4 have discriminated effectively.

In particular, Question One, which had the highest mean score, (see **Table One**) was also the best overall discriminator. However Question Six which had the lowest mean score, was the worst discriminating question overall but, at the same time, was by far the best discriminator for the gold medallist group of contestants.

TABLE THREE:

1988 IMO Mean Scores/Age Group

Age as of 1/7/88	Mean Score + (Number of Contestants)		
	Medallists	Non-Medallists	Overall
12	34 *(1)	-	34.0 (1)
13	-	-	-
14	2 (1)	-	2.0 (1)
15	30.8 (5)	9.0 (4)	21.1 (9)
16	24.6 (21)	8.5 (23)	16.2 (44)
17	24.2 (43)	6.1 (43)	15.1 (86)
18	22.6 (52)	6.8 (52)	14.7 (104)
19	24.8 (8)	6.7 (15)	13.0 (23)

* It is believed that this contestant, Terry Tao from Australia, is the youngest ever in the 29 year history of the IMO to receive a gold medal. Incidentally, he obtained silver and bronze medals at the 1987 and 1986 IMOs respectively.

Table Three shows how well the contestants' younger age groups achieved at the Olympiad in comparison with their older peers where the means ranged from 21.1 to 13.0 for the 15 to 19 age groups.

TABLE FOUR:

1988 IMO Distribution of Awards

Country	Team Size	Code	Score	Medals			Honourable Mention
				Gold	Silver	Bronze	
Algeria	5	ALG	42		1		1
Argentina	3	ARG	23				1
Australia	6	AUS	100	1		1	1
Austria	6	AUT	110	1	1	1	1
Belgium	6	BEL	76			3	1
Brazil	6	BRA	39				2
Bulgaria	6	BUL	144		4	2	
Canada	6	CAN	124	1	1	2	1
Colombia	6	COL	66			3	
Cuba	6	CUB	35				1
Cyprus	6	CYP	21				1
Czechoslovakia	6	CZE	120		2	2	1
Ecuador	1	EQU	1				
Finland	6	FIN	65			2	1
France	6	FRA	128	1	1	3	1

TABLE FOUR CONT'D

1988 IMO Distribution of Awards

Country	Team Size	Code	Score	Medals			Honourable Mention
				Gold	Silver	Bronze	
Federal Republic Of Germany	6	FRG	174	1	4	1	
German Democratic Republic	5	GDR	145	1	4		
Greece	6	GRE	65			1	3
Hong Kong	6	HKG	68			2	1
Hungary	6	HUN	109		2	2	1
Iceland	4	ICE	37			1	
Indonesia	3	INA	6				
Iran	6	IRA	86		1	3	
Ireland	6	IRE	30				
Israel	6	ISA	115	1		4	1
Italy	4	ITA	44			1	1
Kuwait	6	KUW	23				
Luxembourg	3	LUX	64		1	2	
Mexico	6	MEX	40			1	2
Morocco	6	MOR	62			2	1
Netherlands	6	NET	85			3	1
Norway	6	NOR	33				
New Zealand	6	NZL	47		1		
Peru	6	PER	55			1	3
Philippines	5	PHI	29				1
Poland	3	POL	54		1		2
Peoples Republic Of China	6	PRC	201	2	4		
Republic Of Korea	6	ROK	79			3	
Romania	6	ROM	201	2	4		
Singapore	6	SIN	96		2	2	
Spain	6	SPA	34				1
Sweden	6	SWE	115	1		4	1
Tunisia	4	TUN	67			3	
Turkey	6	TUR	65			3	
United Kingdom	6	UNK	121		3	2	
United States	6	USA	153		5	1	
USSR	6	USS	217	4	2		
Peoples Republic Of Vietnam	6	VIE	166	1	4		
Yugoslavia	6	YUG	92			4	1

Table Four summarises the teams' achievements at the 1988 IMO noting the number of gold, silver and bronze medallists and honourable mentions for each team.

TABLE FIVE:

Comparison of the 1981-1988 Results

Olympiad	22nd	23rd	24th	25th	26th	27th	28th	29th
Year	1981	1982	1983	1984	1985	1986	1987	1988
Host Country	USA	Hungary	France	Czechoslovakia	Finland	Poland	Cuba	Australia
Team Size	8	4	6	6	6	6	6	6
# Countries	27	30	32	34	38	37	42	49
# Candidates	185	119	186	192	209	210	237	268
# Total Medallists	*56%* 103	*51%* 61	*50%* 93	*51%* 98	*48%* 101	*51%* 107	*51%* 120	*45%* 120
# Gold Medallists *(range of scores)*	*19%* 36 *(42-41)*	*8%* 10 *(42-37)*	*5%* 9 *(42-37)*	*7%* 14 *(42-40)*	*7%* 14 *(42-34)*	*9%* 18 *(42-34)*	*9%* 22 *(42-42)*	*6%* 17 *(42-32)*
# Silver Medallists *(range of scores)*	*20%* 37 *(40-34)*	*17%* 20 *(36-30)*	*15%* 27 *(36-26)*	*18%* 35 *(39-26)*	*17%* 35 *(33-22)*	*20%* 41 *(33-26)*	*18%* 42 *(41-32)*	*14%* 38 *(31-23)*
# Bronze Medallists *(range of scores)*	*16%* 30 *(33-27)*	*26%* 31 *(29-22)*	*31%* 57 *(25-15)*	*26%* 49 *(25-17)*	*25%* 52 *(21-15)*	*23%* 48 *(25-17)*	*24%* 56 *(31-18)*	*24%* 65 *(22-14)*
# Special Prizes	?	?	?	?	0	1	0	1
# Honourable Mentions	-	-	-	-	-	-	-	33
Average Score	26.6	20.8	15.3	18.7	14.9	18.1	19.9	15.0

TABLE SIX:

Number of 1988 IMO Contestants who were Previous IMO Participants

Year	Category	# Medallists with			# Non-Medallists
		Gold	Silver	Bronze	
1988	**Contestants**	17	48	65	138
	Contestants who were:				
1987	Gold Medallists	5	6		
	Silver Medallists	4	6	5	
	Bronze Medallists		5	5	4
	Non Medallists		1	4	8
	Participants	9	18	14	12
1986	Gold Medallists		1		
	Silver Medallists	3	1	2	
	Bronze Medallists	1			1
	Non-Medallists			2	1
	Participants	4	2	4	2
1985	Gold Medallists				
	Silver Medallists				
	Bronze Medallists		1		
	Non-Medallists				
	Participants		1		
	Average Age	17.3	17.9	17.10	17.10

The statistics in **Table Six** show that 53 (or 20%) of the 1988 IMO contestants had participated at 1987 IMO and, of these, 9 received gold, 18 silver and 14 bronze medals with an additional 4 achieving honourable mentions. Also note that the average age of the gold medallists was 17 years 3 months, silver medallists 17 year 9 months while the overall average age of the contestants was 17 years 10 months.

TABLE SEVEN:

The IMO - The Spread To All Continents

Year	Europe	Asia	North America	South Amercia	Africa	Oceania
1959	7					
1960	5					
1961	6					
1962	7					
1963	8					
1964	8	1				
1965	9	1				
1966	8	1				
1967	12	1				
1968	11	1				
1969	13	1				
1970	13	1				
1971	13	1	1			
1972	12	1	1			
1973	14	1	1			
1974	14	2	2			
1975	14	2	1			
1976	15	1	2			
1977	17	1	2		1	
1978	12	3	2			
1979	18	2	2	1		
1980	*There was no Olympiad in 1980*					
1981	17	1	4	3	1	1
1982	17	5	3	2	2	1
1983	20	3	3	2	3	1
1984	22	3	3	2	3	1
1985	22	7	3	2	3	1
1986	22	6	3	2	3	1
1987	23	6	6	4	2	1
1988	24	11	4	5	3	2

The dominance of European participation in the early years of the IMO is clearly shown. With greater international involvement this dominance has been reduced until in 1988, for the first time in the history of the IMO, more than 50% of the competing countries were non-European. The IMO has truly become a world-wide international annual event.

TABLE EIGHT:

The IMO - A Contest For All Continents

EUROPE	Y	P	H
Romania	1959	29	4
Hungary	1959	28	3
Czechoslovakia	1959	29	3
Bulgaria	1959	29	2
Poland	1959	28	3
USSR	1959	26	3
German D R	1959	27	2
Yugoslavia	1963	25	2
Finland	1965	15	1
Great Britain	1967	21	1
Sweden	1967	21	-
Italy	1967	9	-
France	1967	19	1
Belguim	1969	11	-
Netherlands	1969	18	-
Austria	1970	18	1
Greece	1975	11	-
F R Germany	1977	11	-
Luxembourg	1979	7	-
Spain	1983	6	-
Cyprus	1984	5	-
Norway	1984	5	-
Iceland	1985	4	-
Ireland	1988	1	-

ASIA	Y	P	H
Mongolia	1964	19	-
Vietnam	1974	12	-
Turkey	1978	5	-
Israel	1979	7	-
Kuwait	1982	7	-
Iran	1985	4	-
China	1985	4	-
Hong Kong	1988	1	-
Indonesia	1988	1	-
Philippines	1988	1	-
Korea	1988	1	-
Singapore	1988	1	-

S. AMERICA	Y	P	H
Brazil	1979	9	-
Colombia	1981	8	-
Venezuela	1981	2	-
Uruguay	1987	1	-
Peru	1987	2	-
Argentina	1988	1	-
Ecuador	1988	1	-

N. AMERICA	Y	P	H
Cuba	1971	16	1
USA	1974	14	1
Mexico	1981	3	-
Canada	1981	8	-
Panama	1987	1	-
Nicaragua	1987	1	-

OCEANIA	Y	P	H
Australia	1981	8	1
New Zealand	1988	1	-

AFRICA	Y	P	H
Algeria	1977	8	-
Tunisia	1981	7	-
Morocco	1983	6	-

This Table shows for each IMO participating country, its first year at an Olympiad **(Y)**, the number of times it has participated **(P)** and the number of times it has hosted an IMO **(H)**.

3. Conclusion

In conclusion it should be emphasised that besides the discovering, encouraging and challenging of mathematically gifted school students, the Olympiads aim at the fostering of friendly international relations between teachers and students. It is believed that both these aims were achieved at this year's IMO in Canberra, Australia.

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English version

FIRST DAY

Canberra, July 15, 1988

1. Consider two coplanar circles of radii R and r $(R > r)$ with the same centre. Let P be a fixed point on the smaller circle and B a variable point on the larger circle. The line BP meets the larger circle again at C. The perpendicular l to BP at P meets the smaller circle again at A (if l is tangent to the circle at P then $A = P$).
   (i) Find the set of values of $BC^2 + CA^2 + AB^2$.
   (ii) Find the locus of the midpoint of AB.

2. Let n be a positive integer and let $A_1, A_2, \ldots, A_{2n+1}$ be subsets of a set B. Suppose that
   (a) each A_i has exactly $2n$ elements,
   (b) each $A_i \cap A_j$ $(1 \leq i < j \leq 2n+1)$ contains exactly one element, and
   (c) every element of B belongs to at least two of the A_i.
   For which values of n can one assign to every element of B one of the numbers 0 and 1 in such a way that each A_i has 0 assigned to exactly n of its elements?

3. A function f is defined on the positive integers by

$$\begin{aligned} f(1) &= 1, \quad f(3) = 3, \\ f(2n) &= f(n), \\ f(4n+1) &= 2f(2n+1) - f(n), \\ f(4n+3) &= 3f(2n+1) - 2f(n), \end{aligned}$$

   for all positive integers n.
   Determine the number of positive integers n, less than or equal to 1988, for which $f(n) = n$.

Time: 4.5 hours

Each problem is worth 7 points.

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JULY 9-21

English version

SECOND DAY

Canberra, July 16, 1988

4. Show that the set of real numbers x which satisfy the inequality

$$\sum_{k=1}^{70} \frac{k}{x-k} \geq \frac{5}{4}$$

is a union of disjoint intervals, the sum of whose lengths is 1988.

5. ABC is a triangle right-angled at A, and D is the foot of the altitude from A. The straight line joining the incentres of the triangles ABD, ACD intersects the sides AB, AC at the points K, L respectively. S and T denote the areas of the triangles ABC and AKL respectively. Show that $S \geq 2T$.

6. Let a and b be positive integers such that $ab+1$ divides a^2+b^2. Show that $\dfrac{a^2+b^2}{ab+1}$ is the square of an integer.

Time: 4.5 hours

Each problem is worth 7 points.