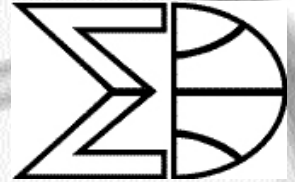


VOLUME 15 NUMBER 1 2002

# MATHEMATICS COMPETITIONS



JOURNAL OF THE  
WORLD FEDERATION OF NATIONAL  
MATHEMATICS COMPETITIONS

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JOURNAL OF THE WORLD FEDERATION OF NATIONAL MATHEMATICS COMPETITIONS

(ISSN 1031 – 7503)

*is published biannually by*

AMT PUBLISHING

UNIVERSITY OF CANBERRA ACT 2601

AUSTRALIA

*Articles (in English) are welcome.*

*Please send articles to:*

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*Mathematics Competitions*

*World Federation of National Mathematics Competitions*

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*The aims of the Federation are:–*

- 1. to promote excellence in, and research associated with, mathematics education through the use of school mathematics competitions;*
- 2. to promote meetings and conferences where persons interested in mathematics contests can exchange and develop ideas for use in their countries;*
- 3. to provide opportunities for the exchanging of information for mathematics education through published material, notably through the Journal of the Federation;*
- 4. to recognize through the WFNMC Awards system persons who have made notable contributions to mathematics education through mathematical challenge around the world;*
- 5. to organize assistance provided by countries with developed systems for competitions for countries attempting to develop competitions;*
- 6. to promote mathematics and to encourage young mathematicians.*

## From the Editor

Welcome to *Mathematics Competitions* Vol 15, No 1.

At the time of writing, preparations are well under way for the WFNMC Congress-4 to be held in Melbourne from 4th to 10th August. There is an excellent range of keynote speakers and we anticipate publishing as many as these as possible in forthcoming issues of the journal. Again, I would like to thank the Australian Mathematics Trust for its continued support, without which the journal could not be published, and in particular Heather Sommariva, Sally Bakker and Richard Bollard of the Trust for their assistance in the preparation of the journal.

### Submission of articles:

The journal *Mathematics Competitions* is interested in receiving articles dealing with mathematics competitions, not only at national and international level, but also at regional and primary school level. There are many readers in different countries interested in these different levels of competitions.

- The journal traditionally contains many different kinds of articles, including reports, analyses of competition problems and the presentation of interesting mathematics arising from competition problems. Potential authors are encouraged to submit articles of all kinds.
- To maintain and improve the quality of the journal and its usefulness to those involved in mathematics competitions, all articles are subject to review and comment by one or more competent referees. The precise criteria used will depend on the type of article, but can be summarised by saying that an article accepted must be correct and appropriate, the content accurate and interesting, and, where the focus is mathematical, the mathematics fresh and well presented. This editorial and refereeing process is designed to help improve those articles which deserve to be published.

At the outset, the most important thing is that if you have anything to contribute on any aspect of mathematics competitions at any level, at your local level or whatever, we would welcome your contribution.

Articles should be submitted in English, with a black and white photograph and a short profile of the author. Alternatively, the article can be submitted on an IBM PC compatible disk or a Macintosh disk. The preferred format is  $\LaTeX$  or  $\TeX$ , but any text file will be helpful.

Articles, and correspondence, can also be forwarded to the editor by mail to

The Editor, *Mathematics Competitions*  
Australian Mathematics Trust  
University of Canberra ACT 2601  
AUSTRALIA

or by email to the address <warrena@amt.canberra.edu.au> or by fax to the Australian Mathematics Trust office, + 61 2 6201 5052, (02 6201 5052 within Australia).

*Warren Atkins,*  
*June 2002*

\* \* \*



## **From the President**

At the time of writing this, the organising committee is busily preparing itself for the upcoming fourth Conference of WFNMC. It does appear that the number of registrations is approaching 80, which is at least as many as the last such conference in Zhong Shan in 1998. The last conference was more strategically located geographically, as it was near the IMO which had just been held in Taiwan. Given that this year's IMO is in Glasgow, I feel that this is a healthy enrolment.

The conference will try to extract the best features of the previous three conferences. As with the first and third conference, all delegates will live in the one hotel, highly enhancing the interactive potential. The problems section will have a slightly new aspect, with each delegate being asked to speak briefly on their favourite problem of the previous year, whether composed by them or not.

A refreshing set of talks has been assembled. John Conway, from Princeton, has agreed to be a keynote lecturer, and a broad representation of speakers have been invited to also present plenary lectures. Some of the topic groups have proved more popular than others, but the talks as submitted, contain a wealth of interesting topics.

There will be a general meeting of the members during the conference to discuss a range of business issues. Two issues which were raised in Tokyo in 2000 will be followed up.

Tony Gardiner has led a task force to investigate methods of increasing the role of teachers. Tony's group has prepared a report and will deliver this in Melbourne. I have also coordinated the writing of a draft policy paper which is designed to articulate the relation between competitions and their related activities with mathematics education in general. This draft paper appeared in an earlier journal, but will be presented to the general meeting for discussion and ratification.

I feel that the WFNMC needs to go through a further step if it is to fully achieve an identity as a widely representative body. At present members of the organisation are individuals interested in competitions and related bodies. Whereas many of these members have contributed

greatly to the success of WFNMC, in many ways they often don't have official status or support within their own countries. Most of the major countries and many smaller ones are represented in the attendance list of the coming conference. However, it would appear that many countries, some major countries included, do not have formal structures reaching out to students of average ability.

Most countries are represented at IMO, although I note that two major countries, Egypt and Pakistan, are not. While it is still a body representative of the international community, in many of these countries, however, the focus of competitions is on the elite. A number of countries have their national Olympiad competition in a number of phases, with the first phase often attracting a large number of entries.

In many of these first phase events, however, the reason for existence is merely to act as an identifier of elite students and filter them into the second stage. The organisers of these events are often focused on the national team and participation in IMO.

It may be that at this stage many countries are losing the obvious opportunity they have to reach out to average students, enrich their lives and to thereby benefit their countries to a greater extent. By greater participation in the WFNMC, with its expertise at this level, such countries may benefit, and through the WFNMC we may all learn a little more.

There may be a method of enabling this, by encouraging membership of national organisations. A similar organisation for physics was recently established, and their membership is based more on national organisations. I am not suggesting that individual membership of WFNMC should cease, but that we explore methods, maybe by corporate national memberships, of extending the international flavour of WFNMC to give even wider coverage.

*Peter Taylor*  
*President*

\* \* \*

## **WFNMC Congress 4 – Melbourne**

**August 4 - 11 2002**

The WFNMC-4 Congress will be held in Melbourne Australia, from Sunday August 4 to Sunday August 11 2002. The conference will be centrally located in Melbourne. The Congress is following the pattern of previous Congresses instigated at Waterloo, where all participants are located at the one venue and a single registration fee will cover all costs, accommodation, meals, transport and the excursion. There will be a program for accompanying persons.

An excellent program of plenary speakers has been arranged and the main topic areas will enable participants to discuss aspects of interesting problems they have come across.

The proposed program for this Congress contains eight Plenary Lectures, a Main Topic Area and six other topic areas. A description of the anticipated Congress program follows:-

### **Program**

#### **Plenary Lectures**

1. Anne Street (prominent Australian mathematician)
2. John Conway
3. Alexander Soifer & David Coulson
4. Kaye Stacey (prominent Australian mathematics educator)
5. Andy Liu

6. Andre & Jean Christophe Deledicq
7. Robert Geretschläger
8. Petar Kenderov (overview)

It is anticipated that all topics will be of general mathematical or problem solving interest, with the exception that Petar Kenderov will present an overview (state of the nation!) of the WFNMC.

### **Main Topic Area**

Each participant will be asked to present and discuss his/her favourite problem (not necessarily theirs) of recent years in approx 15 minutes to the particular problem group. Each participant will be asked to submit his or her selected problem and solution before the Congress.

It is planned to have the participants receive a copy of the problems and solutions before each session so that the focus of these sessions can be on interesting aspects of the problem such as the creation of the problem, student reaction to and performance on the problem and interesting aspects of the solutions.

There will be three parallel sessions, relating to the three competition level:-

- national level
- senior inclusive ( $\geq$  grade 10)
- junior inclusive ( $\leq$  grade 9)

Jaroslav Svrcek, (Czech Republic), Bill Richardson (United Kingdom) and Bruce Henry (Australia), respectively, are chairs of these topic areas.

## **Topic Areas**

There are six Topic Areas. They and their conveners are:-

1. Geometry Ali Rejali (Iran)
2. Research related to Competitions Maria deLosada (Colombia)
3. The Creation of Olympiad Problems Robert Geretschläger  
(Austria)
4. Inclusive Competitions (eg Canadian, Australian, Kangarou)  
Peter Crippin (Canada)
5. The Involvement of Teachers in Competitions and the Role of  
Competitions in Mathematics Education  
Tony Gardiner (United Kingdom)
6. Informatics Competitions David Clark (Australia)

A summary of the program follows:-

## Congress Program

August	5	6	7	8	9	10
0900 to 1000	Opening & Plenary 1	Plenary 3	Main	Plenary 4	Plenary 6	Plenary 8
1000 to 1100		Main T. A.		Topic Area 1	Main T. A.	WFNMC Meeting & Reports Lunch
Break	Main T. A.	Main T. A.		Plenary 5 1200-1300	Main T. A.	
1130 to 1300		Lunch				
1400 to 1530	T. A.	Teachers & Competitions <sup>†</sup>	Excursion	T. A. 3 & 4	Plenary 7	Cultural and/or Football Outings
Break 1600 to 1700.		T. A.			T of T Meeting	
	1 & 2	5 & 6				
1730 to 1830	Ceremony & Plenary 2	Drinks	MAV Function	Drinks	Bus for	Drinks
1830 to 1930	Dinner	Dinner		Dinner	Dinner	Closing Dinner

† Report and discussion on the report: *Teachers, Competitions and Mathematics Education*  
Tony Gardiner (UK)

The John Conway plenary and the welcoming ceremony will be on Monday afternoon at 1730.

\* \* \*

## Letter

I would like to refer to the proof of the statements (A) and (C) in the article

Some properties of functions of the form  $f(x) = \frac{x^2+ax+b}{x^2+cx+d}$

by Xin Li and Andy Liu, *Mathematics Competitions*, **14 2** 2001.

Let  $f(x) = \frac{x^2+ax+b}{x^2+cx+d}$  with  $(a-c)^2 + (b-d)^2 \neq 0$ . Prove that two following statements are equivalent:

a) There exist such real numbers  $h < k$ , so that for each  $x$  (in the domain of  $f$ ) either  $f(x) < h$  or  $f(x) > k$

b)  $(b-d)^2 + (a-c)(ad-bc) > 0$ .

Note that the condition a) is equivalent to the statement, that the equation  $f(x) - 1 = A$  does not have roots when  $A \in [h-1, k-1]$ , that is the discriminant  $D$  of the equation  $Ax^2 + (Ac - (a-c))x + Ad - (b-d) = 0$  is less than zero ( $D < 0$ ) for  $A$  belonging to some domain. This statement, in its turn, is equivalent to a statement, that the nonempty solution of inequality  $D = (c^2 - 4d)A^2 + 2(2(b-d) - c(a-c))A + (a-c)^2 < 0$  exists.

Let us now prove, that if  $C_1 \geq 0, B_1^2 + C_1^2 \neq 0$ , then the inequality

$$A_1 t^2 + B_1 t + C_1 < 0 \tag{1}$$

will have a solution if and only if  $D = B_1^2 - 4A_1C_1 > 0$ .

Indeed, (1) has a solution when  $A_1 < 0$ , or  $A_1 = 0, B_1 \neq 0$ , or  $A_1 > 0, D > 0$ .

In all cases  $D > 0$  (the case of  $A_1 < 0, C_1 = B_1 = 0$  is impossible).

Now when  $D = B_1^2 - 4A_1C_1 > 0$ , the following cases are possible:

$A_1 < 0$ , or  $A_1 = 0, B_1 \neq 0$ , or  $A_1 > 0$ .

In all these cases (1) has a nonempty solution.

Thus the condition a) is equivalent to the condition that

$$\frac{D}{4} = (2(b-d) - c(a-c))^2 - (c^2 - 4d)(a-c)^2 > 0$$

or

$$(b-d)^2 + (a-c)(ad-bc) > 0.$$

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\* \* \*



## The Transformation of Mutual Reciprocal Roots and Its Applications

*Li Jinghua & Zhang Junda*



*Zhang Junda is a professor and director of the Institute of Educational Science, Capital Normal University, Beijing. He is actively involved in the Beijing Mathematical Olympiad, the Beijing and Chinese Mathematical Societies and the Youth Councilors Association for Science and Technology. He is also involved with the Primary and Middle School Olympiads.*

*Li Jinghua is a PhD student and lecturer at the Institute of Systems Science, Academy of Mathematics and System Science, the Chinese Academy of Sciences. He has published more than 10 research papers and 6 text books.*

### Abstract

In this paper, we will construct a transformation of mutual reciprocal roots by using a transit matrix between the two bases of the vector space according to algebraic theory. It can be applied to solve a typical kind of problem of algebra, and in particular, it can be used to solve an International Mathematical Olympiad problem.

## 1 Lemma

According to theory of algebra [2], [3] it is easy to imply the following conclusions:

1.  $\{1, x + x^{-1}, x^2 + x^{-2}, \dots, x^{n-1} + x^{-(n-1)}\}$  and  $\{1, x + x^{-1}, (x + x^{-1})^2, \dots, (x + x^{-1})^{n-1}\}$  are two bases of an  $n$ -dimensional vector space respectively, where  $x \in V$ ,  $V$  denotes  $n$ -dimensional vector space.
2. Suppose that  $x^i + x^{-i} = B_i$ , where  $B_0 = 1$ ,  $(x + x^{-1})^i = A_i$ ,  $i = 0, 1, 2, \dots, n - 1$ .

We can calculate the transit matrix from the base  $\{A_0, A_1, A_2, \dots, A_{n-1}\}$  to the base  $\{B_0, B_1, B_2, \dots, B_{n-1}\}$ , that is  $\{A_0, A_1, A_2, \dots, A_{n-1}\} = \{B_0, B_1, B_2, \dots, B_{n-1}\}T$

## 2 Main ideas

**Definition** Suppose that  $F$  denotes a number field,  $V$  denotes an  $n$ -dimensional vector space. If  $\sigma$  is the map from  $V$  to  $V$  determined by,

$$\sigma : \xi \mapsto \eta,$$

in which

$$\xi = (B_0, B_1, B_2, \dots, B_{n-1}) \begin{pmatrix} a_0 \\ a_1 \\ \vdots \\ a_{n-1} \end{pmatrix},$$

$$\eta = (A_0, A_1, A_2, \dots, A_{n-1})T \begin{pmatrix} a_0 \\ a_1 \\ \vdots \\ a_{n-1} \end{pmatrix}.$$

We say that,  $\sigma$  is the transformation of mutual reciprocal roots [1].

**Theorem 1**

The transformation of mutual reciprocal roots is a linear transformation.

**Proof**

(i) Suppose that

$$\xi_1 = (B_0, B_1, B_2, \dots, B_{n-1}) \begin{pmatrix} a_0 \\ a_1 \\ \cdot \\ \cdot \\ a_{n-1} \end{pmatrix},$$

$$\xi_2 = (B_0, B_1, B_2, \dots, B_{n-1}) \begin{pmatrix} b_0 \\ b_1 \\ \cdot \\ \cdot \\ b_{n-1} \end{pmatrix}$$

are two arbitrary vectors of the vector space  $V$ , we have

$$\sigma(\xi_1 + \xi_2) =$$

$$\sigma \left( (B_0, B_1, \dots, B_{n-1}) \begin{pmatrix} a_0 \\ a_1 \\ \cdot \\ \cdot \\ a_{n-1} \end{pmatrix} + (B_0, B_1, \dots, B_{n-1}) \begin{pmatrix} b_0 \\ b_1 \\ \cdot \\ \cdot \\ b_{n-1} \end{pmatrix} \right)$$

equals

$$\sigma \left( (B_0, B_1, B_2, \dots, B_{n-1}) \begin{pmatrix} a_0 + b_0 \\ a_1 + b_1 \\ \vdots \\ a_{n-1} + b_{n-1} \end{pmatrix} \right)$$

which equals

$$(A_0, A_1, A_2, \dots, A_{n-1}) T \begin{pmatrix} a_0 + b_0 \\ a_1 + b_1 \\ \vdots \\ a_{n-1} + b_{n-1} \end{pmatrix}$$

which equals

$$(A_0, A_1, \dots, A_{n-1}) T \begin{pmatrix} a_0 \\ a_1 \\ \vdots \\ a_{n-1} \end{pmatrix} + (A_0, A_1, \dots, A_{n-1}) T \begin{pmatrix} b_0 \\ b_1 \\ \vdots \\ b_{n-1} \end{pmatrix}$$

$$= \sigma(\xi_1) + \sigma(\xi_2)$$

(ii) Assume that  $a \in F$  and

$$\xi = (B_0, B_1, B_2, \dots, B_{n-1}) \begin{pmatrix} a_0 \\ a_1 \\ \vdots \\ a_{n-1} \end{pmatrix} \in V,$$

$$\begin{aligned}\sigma(a\xi) &= \sigma \left( a(B_0, B_1, B_2, \dots, B_{n-1}) \begin{pmatrix} a_0 \\ a_1 \\ \cdot \\ \cdot \\ a_{n-1} \end{pmatrix} \right) \\ &= \sigma \left( (B_0, B_1, B_2, \dots, B_{n-1}) \begin{pmatrix} aa_0 \\ aa_1 \\ \cdot \\ \cdot \\ aa_{n-1} \end{pmatrix} \right) \\ &= (A_0, A_1, A_3, \dots, A_{n-1})T \begin{pmatrix} aa_0 \\ aa_1 \\ \cdot \\ \cdot \\ aa_{n-1} \end{pmatrix} \\ &= a(A_0, A_1, A_3, \dots, A_{n-1})T \begin{pmatrix} a_0 \\ a_1 \\ \cdot \\ \cdot \\ a_{n-1} \end{pmatrix} \\ &= a\sigma(\xi)\end{aligned}$$

Hence  $\sigma$  is a linear transformation.

Regarding the transit matrix  $T$ , we have

$$T = \begin{pmatrix} 1 & 0 & -2 & 0 & 2 & 0 & -2 & \dots \\ 0 & 1 & 0 & -3 & 0 & 5 & 0 & \dots \\ 0 & 0 & 1 & 0 & -4 & 0 & 9 & \dots \\ 0 & 0 & 0 & 1 & 0 & -5 & 0 & \dots \\ 0 & 0 & 0 & 0 & 1 & 0 & -6 & \dots \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & \dots \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & \dots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \end{pmatrix}$$

where

- (i)  $a_{11} = 1$ ,  $a_{i1} = 0$  for  $i \geq 2$   
 $a_{12} = 0$ ,  $a_{22} = 1$ ,  $a_{i2} = 0$  for  $i \geq 3$   
 $a_{1i} = 0$ , for  $i = 2m$ ,  $m = 1, 2, 3, \dots$

$a_{1j} = (-1)^{\lfloor \frac{j}{2} \rfloor} 2$ ,  $j = 2m + 1$ ,  $m = 1, 2, \dots$ , where  $\lfloor i \rfloor$  denotes the largest integer not greater than  $i$ .

- (ii) Besides the laws mentioned above for the 1st row and the 1st and 2nd column, the elements of any other position are given by

$$a_{ij} = a_{i-1,j-1} - a_{i,j-2} \text{ for } i \geq 2, j \geq 3.$$

In fact, the transit matrix  $T$  demonstrates the above-mentioned rule, and following reference [1], we take out 0 among non-zero elements of the upper trigonometric matrix then

$$T' = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & \dots \\ 0 & 1 & -2 & 0 & 0 & 0 & 0 & \dots \\ 0 & 0 & 1 & -3 & 2 & 0 & 0 & \dots \\ 0 & 0 & 0 & 1 & -4 & 5 & -2 & \dots \\ 0 & 0 & 0 & 0 & 1 & -5 & 9 & \dots \\ 0 & 0 & 0 & 0 & 0 & 1 & -6 & \dots \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & \dots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \end{pmatrix}.$$

We can give the general form of any element  $a_{ij}$  of the matrix  $T'$  at the  $i$ th row and  $j$ th column

$$a_{ij} = (-1)^{j-i} (c_i^{j-i} - c_{i-2}^{j-i-2})$$

where we define  $c_m^n = 0$  when  $n < 0$  and  $c_m^n = 0$  when  $m < n$ , and, in particular,  $a_{12} = 0$ .

**Theorem 2** The transformation of mutual reciprocal roots is a reversible transform.

By the assumption in the definition, it is obvious that  $\sigma$  is an identical transformation, so the transformation of mutual reciprocal roots is a reversible transform

**Corollary** The vector which can be expressed by the base  $\{B_0, B_1, B_2, \dots, B_{n-1}\}$  and the base  $\{A_0, A_1, A_2, \dots, A_{n-1}\}$  can be mutually transformed by the transformation of mutual reciprocal roots.

We construct the transformation of mutual reciprocal roots, because it can be widely applied to a kind of representative problem of algebra.

### 3 Applications

**Example 1** Solve the equation

$$2x^8 - 17x^7 + 30x^6 + 17x^5 - 64x^4 + 17x^3 + 30x^2 - 17x + 2 = 0.$$

**Solution** This equation can be transformed using the base

$$\{1, x + x^{-1}, \dots, x^4 + x^{-4}\}$$

to get

$$-64 + 17(x + x^{-1}) + 30(x^2 + x^{-2}) - 17(x^3 + x^{-3}) + 2(x^4 + x^{-4}) = 0.$$

Apply the transformation  $\sigma$  and let  $y = x + x^{-1}$ :

We have

$$(1, y, y^2, y^3, y^4) T \begin{pmatrix} -64 \\ 17 \\ 30 \\ -17 \\ 2 \end{pmatrix} = 0,$$

where

$$T = \begin{pmatrix} 1 & 0 & -2 & 0 & 2 \\ 0 & 1 & 0 & -3 & 0 \\ 0 & 0 & 1 & 0 & -4 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

Expanding and factorizing, we obtain

$$(y^2 - 8y + 12)(2y^2 - y - 10) = 0$$

This gives

$$y_1 = 2, y_2 = 6, y_3 = \frac{5}{2}, y_4 = -2.$$

From this, we have

$$x_1 = 1, x_2 = 1, x_3 = 3 + 2\sqrt{2}, x_4 = 3 - 2\sqrt{2}$$

$$x_5 = 2, x_6 = \frac{1}{2}, x_7 = -1, x_8 = -1.$$

We can solve some problems of multiple angles in trigonometry by using the transformation of mutual reciprocal roots:

**Example 2** For any natural number  $n$ , express  $\cos n\alpha$  as a function of  $\cos \alpha$ .

**Solution** Let  $x = \cos \alpha + i \sin \alpha$



Then

$$\begin{aligned}
 x^{-1} &= \cos \alpha - i \sin \alpha \\
 2 \cos n\alpha &= x^n + x^{-n} \\
 &= (1, x + x^{-1}, \dots, x^n + x^{-n}) \begin{pmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{pmatrix}
 \end{aligned}$$

Applying  $\sigma$  we obtain

$$2 \cos n\alpha = (1, x + x^{-1}, \dots, (x^n + x^{-1})^n) T \begin{pmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{pmatrix}.$$

Since  $x + x^{-1} = 2 \cos \alpha$ , we have then obtained a polynomial in  $\cos \alpha$ .

In quantum chemistry, there is often a need to do similar calculations.  
[4]

**Example 3** Compute the determinant

$$D_n = \begin{vmatrix} 2 \cos \alpha & 1 & 0 & \cdots & 0 \\ 1 & 2 \cos \alpha & 1 & \cdots & 0 \\ 0 & 1 & 2 \cos \alpha & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 2 \cos \alpha \end{vmatrix}$$

**Solution** Let  $x = \cos \alpha + i \sin \alpha$

Then  $x^{-1} = \cos \alpha - i \sin \alpha$

and

$$D_n = \begin{vmatrix} x + x^{-1} & 1 & 0 & \cdots & 0 \\ 1 & x + x^{-1} & 1 & \cdots & 0 \\ 0 & 1 & x + x^{-1} & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & x + x^{-1} \end{vmatrix}$$

From this we can deduce

$$D_n = (1, x + x^{-1}, \dots, (x^n + x^{-1})^n) \begin{pmatrix} 0 \\ \vdots \\ 0 \\ (-1)^r c_{n-r}^r \\ \vdots \\ (-1)^2 c_{n-2}^2 \\ 0 \\ (-1)^1 c_{n-1}^1 \\ 0 \\ (-1)^0 c_{n-0}^0 \end{pmatrix}$$

for  $(0 \leq r \leq \lfloor \frac{n}{2} \rfloor)$ .

Applying the transformation  $\sigma^{-1}$  we have

$$\begin{aligned} D_n &= (1, x + x^{-1}, \dots, x^n + x^{-1}) T^{-1} \begin{pmatrix} 0 \\ \vdots \\ (-1)^r c_{n-r}^r \\ \vdots \\ 1 \end{pmatrix} \\ &= \frac{x^{n+1} - x^{-(n+1)}}{x - x^{-1}} \\ &= \frac{\sin(n+1)\alpha}{\sin \alpha} \end{aligned}$$

Finally, we shall consider a problem from the 18th IMO:

**Example 4** Let  $p_1(x) = x^2 - 2$  and  $p_i(x) = p_1[p_{i-1}(x)]$ ,  $i = 1, 2, 3, \dots$ . Prove that: for any natural number  $n$ , the roots of the equation  $p_n(x) = x$  are all different real roots.

**Proof** The interval  $[-2, 2]$  is mapped onto the interval  $[-2, 2]$  by the function  $p_1$ . When  $x$  increases from  $-2$  to  $0$ ,  $p_1(x)$  decreases from  $2$  to  $-2$ , and as  $p_1(x)$  is an even function, when  $x$  increases from  $0$  to  $2$ ,  $p_1(x)$  increases from  $-2$  to  $2$ . As  $p_2(x) = p_1[p_1(x)]$ , when  $x$  changes from  $-2$  to  $0$ ,  $p_1(x)$  changes from  $2$  to  $-2$ . Similarly,  $p_2(x) = p_1[p_1(x)]$  has the range  $[-2, 2]$ .

Since  $p_n(x) = x$ , then  $p_n(x)$  should be considered on  $[-2, 2]$  for any natural number  $n$ .

Let  $2 \cos \alpha = x$  and  $\alpha \in [0, \pi]$ , then  $x \in [-2, 2]$ .

Consider  $x = \cos \alpha + i \sin \alpha$  as  $z + z^{-1}$ , where

$$z = \cos \alpha + i \sin \alpha, \quad z^{-1} = \cos \alpha - i \sin \alpha.$$

Then,

$$p_1(x) = p_1(2 \cos \alpha) = p_1(z + z^{-1}) = z + z^{-2} = 2 \cos 2\alpha.$$

In general,

$$p_n(x) = p_n(z + z^{-1}) = z^{2^n} + z^{2^{-n}} = 2 \cos 2^n \alpha.$$

Thus the equation  $p_n(x) = x$  is transformed into

$$2 \cos 2^n \alpha = 2 \cos \alpha$$

It follows that from this equation

$$2^n \alpha = \pm \alpha + 2k\pi, \quad k = 0, 1, 2, 3, \dots,$$

that

$$\alpha = \frac{2k\pi}{2^n - 1} \quad \text{and} \quad \alpha = \frac{2k\pi}{2^n + 1}$$

As  $\alpha \in [0, \pi]$ , then for  $k = 0, 1, 2, 3, \dots, 2^{n-1} - 1$ , the first expression for  $\alpha$  gives  $2^{n-1}$  different values of  $\cos \alpha$ .

For  $k = 0, 1, 2, 3, \dots, 2^{n-1}$ , the second expression for  $\alpha$  gives another  $2^{n-1}$  different values of  $\cos \alpha$ .

Hence, we obtain  $2 \times 2^{n-1} = 2^n$  different real roots of  $x = 2 \cos \alpha$  satisfying the equation  $p_n(x) = x$ .

Therefore,

$$x = 2 \cos \frac{2k\pi}{2^n - 1}, \quad k = 0, 1, 2, 3, \dots, 2^{n-1} - 1$$

and

$$x = 2 \cos \frac{2k\pi}{2^n + 1}, \quad k = 0, 1, 2, 3, \dots, 2^{n-1}$$

are  $2^n$  different real roots of the equation  $p_n(x) = x$ .

$p_n(x) = [\dots((x-2)^2 - 2)^2 \dots]^2 - 2$  is a  $n$ -degree polynomial, and  $p_n(x) = x$  is a  $2^n$ -degree equation. According to the basic theorem of algebra, the equation  $p_n(x) = x$  has only these  $2^n$  different real roots.

In addition, for the Chebyshev polynomial

$$T_n(x) = \frac{1}{2^n} \left[ \left( x + \sqrt{x^2 - 1} \right)^n + \left( x - \sqrt{x^2 - 1} \right)^n \right] \quad |x| \leq 1,$$

let  $x = \cos \alpha$ ,  $\sqrt{x^2 - 1} = i \sin \alpha$ .

Applying the transformation of mutual reciprocal roots, the following conclusions are readily obtained:

- (i)  $T_n(x)$  is a  $n$ -degree polynomial in  $x$  with the first coefficient 1;
- (ii)  $T_n(x)$  has only  $n$  different real roots;
- (iii) The following identity holds:

$$T_n(x) - xT_{n-1}(x) + \frac{1}{4}T_{n-2}(x) \equiv 0$$

- (iv) There is always a root of  $T_{n-1}(x)$  between two neighbouring roots of  $T_n(x)$ .

It can be seen that Example 4 is a special case of the Chebyshev Polynomial  $T_2(2x)$ .

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## The Hunter Primary Mathematics Competition – 21 Years On

*Fred Bishop*



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The forerunner to this article, which was published in this journal in 1988, provided a general outline as to how the Hunter Primary Mathematics Competition was established and administered. In this sequel, it can be reported that this competition for Year 5 and 6 (i.e. 10 to 12 year old) students has continued to receive excellent support from primary schools across the Hunter Region. To indicate the growth in support for this competition, the participation figures for students and schools for 1981 and then for the 1997 - 2001 period are provided in Table 1. It is extremely pleasing to report that the 2001 competition had record student entries. The school participation figures for the last five years suggest that support from Hunter primary schools has been sustained throughout this period.

### Student and School Participation Figures 1981: 1997 2001

Student Participation					
Year	Year 5		Year 6		Total Students
	Boys	Girls	Boys	Girls	
1981	576	521	648	592	2337
1997	3823	3573	3739	3755	14890
1998	3520	3598	3962	3714	14794
1999	3674	3648	3854	3800	14976
2000	3686	3618	3628	3619	14551
2001	4014	3953	4001	3998	15966

School Participation				
Year	State	Catholic	Independent	Total
1981	49	19	2	70
1997	196	41	15	252
1998	194	42	14	250
1999	194	42	14	250
2000	183	42	13	238
2001	185	43	14	242

For the past twenty-one years this competition has enjoyed the whole-hearted support of its sponsors, the Newcastle Permanent Building Society, and hundreds of teachers who have ensured the successful administration of this exercise at the school level. Five of the six foundation members of the competition's Organising Committee are still actively involved in the management of the competition. In recent years, three Mathematics Consultants from local Education Districts, two members from the University of Newcastle's Department of Mathematics and a few young and highly motivated primary teachers have provided the essential additional support required to cope with the competition's development.

The primary aims of the competition are to:

- identify mathematically talented children within the Hunter Region.
- encourage the broad and in-depth coverage of the New South Wales K-6 Mathematics Curriculum.

On the first Wednesday in September, candidates from Year 5 and 6 sit for a common 45 minute test which requires the answering of 35 graded multiple choice questions. The current paper format contains three sections - Section A (15 basic test items each worth 2 marks), Section B (10 considerably more demanding test items each worth 3 marks) and Section C (10 difficult test items each worth 4 marks). Candidates code in their answers on a mark sense answer card.

Each candidate receives a Participation Certificate on the day the test paper is administered. Late in October, a comprehensive Competition Report is forwarded to schools that have participated with their student results. High Distinction, Distinction or Merit Certificates are also forwarded to participating schools at this time to recognise the commendable performance of the top 45% in each student division of the competition. Major and Consolation Prizes are awarded to the top ten placed students in each division at a formal presentation ceremony in November.

The 100 top placed candidates in the Year 5 Division of each year's competition are invited to attend a weekend mathematics camp which is conducted early in the subsequent school year at a local Department of Sport and Recreation Camp Centre. A program providing a range of varied mathematics workshop sessions for groups of twenty talented students is conducted. To add balance to the weekend's program, each workshop session is followed by a short recreation activity. After the Saturday evening barbecue, a mathematics relay competition is conducted prior to the staging of an entertaining games and social program. All camp programs have been extremely well received.

As the workshop supervisors at these camps are members of the competition's Problem Setting Committee, they are able, at a camp's conclusion, to assess as to how challenging the next competition's test paper should be. This committee believes that it is important to



set a test paper which avoids a clustering of tied scores at the top end of the distribution of marks while, at the same time, enabling the competition's potential winner to have a chance of achieving a maximum score. Table 2 shows the frequency of the number of questions answered incorrectly by the competition winners over the 21 years of the competition's operation. An examination of this table suggests that the Problem Setting Committee's aim to set an appropriately challenging test instrument has been satisfied.

Table 2

**Frequency of Number of Test Items Answered Correctly by Competition Winners**

No. of Test Items Answered Incorrectly by Competition Winners	$f$
0	4
1	8
2	7
3	2

The details in Table 3 illustrates reveal how the format of the test paper has changed since the competition began in 1981. The changes have resulted from the analyses of data obtained from surveys conducted with participating schools.

Table 3

**Changes in Test Format**

Year	No. of Test Items per Section			Test Item Value in Each Section			Total Marks
	Sec A	Sec B	Sec C	Sec A	Sec B	Sec C	
1981	10	10	10	3	4	5	120
1982-1985	10	10	10	2	3	4	90
1986-2001	15	10	10	2	3	4	100

Prior to 1993, a penalty mark system, was used to rank student performance. This system involved deducting one-third of a test item's mark for each incorrectly answered item. In a survey circulated to

schools late in 1992, the majority of participating schools requested that the penalty mark system for ranking students be abandoned. The popular expression supporting this request was the view that young students should not be discouraged to attempt test items. An analysis of the difference in student rankings was consequently conducted when the answer sheets were computer processed for the 1992 competition. This analysis revealed that the difference in student rankings for “the penalty mark” and “no penalty mark” systems was insignificant. As a result, the “penalty mark” system was eliminated from 1993 onwards.

When setting a test paper, the Problems Setting Committee likes to include the odd problem which has topical relevance. As there was much focus, in 2000, on the staging of the Sydney Olympics, the following test items were posed in that year’s test paper.

Question 7 (Section A - worth 2 marks)

The 2000 Olympics conducted in Sydney was the 24th Summer Olympic Games to have been staged in the modern era. If future Summer Olympic Games are to be staged in leap years, the Games in the year 2004 will be the:

- |                          |                          |
|--------------------------|--------------------------|
| (A) 29th Summer Olympics | (B) 30th Summer Olympics |
| (C) 47th Summer Olympics | (D) 48th Summer Olympics |

Question 28 (Section C - worth 4 marks)

In 1999, Ian Thorpe broke the world record for 200 metres freestyle by swimming a time of 1 minute 46.00 seconds. He swam the first hundred metres in 55.01 seconds. How long did it take him to swim the second 100 metres?

- |                   |                            |
|-------------------|----------------------------|
| (A) 51.99 seconds | (B) 50.59 seconds          |
| (C) 50.99 seconds | (D) 1 minute 30.99 seconds |

As the GST (Goods and Services Tax) had been recently introduced and to recognise the passing of Sir Donald Bradman, one of Australia’s top sportsman ever, two topical test items set in the 2001 test paper were:

Question 22 (Section B - worth 3 marks)

The price of a shirt after adding the 10% GST charge was \$55.00.  
What would be the price of this shirt before the GST was added?

- (A) \$60.50      (B) \$60.00      (C) \$50.00      (D) \$49.50

Question 23 (Section B - worth 3 marks)

The table shows the number of test matches won, lost, drawn or tied when Sir Donald Bradman captained the Australian cricket team.

Bradman's Test Record as Captain			
Won	Lost	Drawn	Tied
15	3	6	0

What percentage of matches was won when Sir Donald Bradman captained the team?

- (A) 15% (B) 24% (C) 60% (D) 62.5%

A further consideration of the Problems Setting Committee when setting the test is to include items which cover the range of content areas outlined in the NSW K-6 Mathematics Curriculum. When setting the 'challenge' problems for Section C of a paper, the committee members exercise their licence to pose questions which require candidates to:

- think laterally at the boundaries of the curriculum topics,
- reveal an ability to comprehend technical language in mathematical settings,
- apply 'clever' and efficient techniques to solve problems quickly,
- apply a real understanding of the mathematical principles underlying these techniques.

The items from the 2001 test paper are listed below to illustrate the Problem Setting Committee's intention to include items which satisfy the above objectives.

The date 280201 represents 28 February, 2001. Entries in the Balance column for the dates 310501, 300601 and 310701 did not print in Lisa's Newcastle Permanent Savings Passbook. What balance should have been shown for 31 July, 2001?

DATE	DEPOSIT	WITHDRAWAL	BALANCE
280201			\$640.75
310301	\$15.50		\$656.25
300401		\$23.00	\$633.25
310501		\$ 7.80	
300601	\$18.95		
310701		\$ 9.20	
310801	\$11.50		\$646.70

- (A) \$621.40      (B) \$635.20      (C) \$655.90      (D) \$658.20

---

A rectangular paddock is 75 metres wide. Its area is 3.75 ha. What is the length of this paddock?

- (A) 5 m      (B) 50 m      (C) 500 m      (D) 5000 m

---

$2^{10} = 1024$ . Which one of the following statements is correct?

- (A)  $1\,000\,000 = 1000 \times 2^{10}$  to the nearest hundred  
(B)  $1\,000\,000 = 1000 \times 2^{10}$  to the nearest thousand  
(C)  $1\,000\,000 = 1000 \times 2^{10}$  to the nearest ten thousand  
(D)  $1\,000\,000 = 1000 \times 2^{10}$  to the nearest hundred thousand
-

A box holds red and blue disks. A number is written on each disk. The total of the numbers on these disks is 60. The average of the numbers on the 6 red disks is 4. The average of the numbers on the blue disks is 9. How many blue disks are there in the box?

- (A) 4                      (B) 5                      (C) 6                      (D) 10
- 

A cyclist travelled at 20 km/h for 40 km. This cyclist then travelled at 40 km/h for the next 20 km. The cyclist's average speed for the total journey was:

- (A) 20 km/h      (B) 24 km/h      (C) 30 km/h      (D) 34 km/h
- 

Ten children each donate a different amount to a charity. The least of these amounts is \$12.85 and the greatest is \$13.45. One of the following amounts is the sum of their ten donations. Which of the following must be that amount?

- (A) \$125.60      (B) \$128.50      (C) \$131.60      (D) \$135.
- 

An example of seven consecutive natural numbers is 3, 4, 5, 6, 7, 8 and 9. The sum of the consecutive natural numbers starting at 31 and finishing at 76 is:

- (A) 2441              (B) 2451              (C) 2461              (D) 2471
-

If the pattern shown in Figures 1, 2 and 3 was extended to show Figures 4 and 5, how many shaded squares would there be in Figure 5?



Figure 1



Figure 2

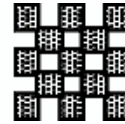
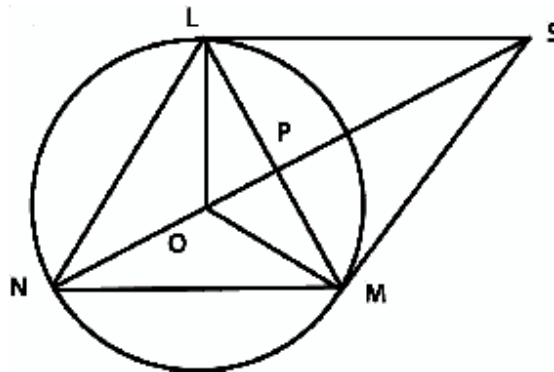


Figure 3

- (A) 39                      (B) 41                      (C) 43                      (D) 45

The points  $L$ ,  $M$  and  $N$  are on the circumference of the circle with centre  $O$ .  $LMN$  is an equilateral triangle.  $LM$  and  $NS$ , the diagonals of the rhombus  $LSMN$ , intersect at  $P$ .



The area of the triangle  $LPO$  is  $6 \text{ cm}^2$ .

What is the area of the triangle  $LSP$ ?

- (A)  $12 \text{ cm}^2$               (B)  $15 \text{ cm}^2$               (C)  $18 \text{ cm}^2$               (D)  $24 \text{ cm}^2$

$12 \times 13 \times 14 \times 15 \times 16$  equals:

- (A) 524 160      (B) 524 320      (C) 524 340      (D) 524 460
- 

The Organising Committee is confident that the current level of support and interest in the Hunter Primary Mathematics Competition will be maintained. This will only be possible, however, with the continued support of the Newcastle Permanent Building Society and the primary schools from across the Hunter Region.

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## Mathematics Challenge at the University of Western Ontario

A Programme of Mathematics Enrichment  
for Students from Grades 5 to School Leaving

*Tom Griffiths*



*Tom Griffiths studied at Imperial College, London, UK and Waterloo, Canada. He has had extensive experience teaching mathematics to first year university level. He has had a long association with mathematics competitions including the US and Canadian Olympiads as well as the Descartes Competition and the Canadian Invitational Challenge. Tom has also been involved in many facets of mathematics education and curriculum construction.*

I was a high school mathematics teacher for 34 years and retired in 1995. During my career I had considerable involvement with mathematics contests, both in preparing students, setting the papers and marking the solutions. I have been involved in various roles in the U.S. Olympiad, the Canadian Olympiad, the Canadian Mathematics Competitions Descartes and Euclid contests and the Canadian Invitational Challenge as Chair from its inception in 1988. When I retired it seemed a shame not to make use of this experience for the benefit of excellent mathematics students. I also wanted to give local students the opportunity to receive the appropriate coaching to work towards becoming members of the Canadian International Mathematics Olympiad (IMO) Team. I had been coach of the team to Finland in 1985, and have had one student on the team to Australia in 1988.

In January of 1996 I started visiting local schools on a weekly basis to help those students who were interested in taking a challenge to excel



in mathematics. For the rest of that academic year I visited several local high schools weekly, for the sake of the few in each school who were interested. However, this turned out to be a very tiring experience, requiring much more effort than benefit warranted. Hence, I reconsidered my strategy and decided that it would be to everyone's benefit, especially mine, if all the students came to me. As it happened, the staff of the local University, the University of Western Ontario, (U.W.O.) decided at the same time that they needed to establish an outreach programme for high ability mathematics students. Fortunately, we managed to discover our mutual goals and agree to combine our experience, and I was delighted that they were willing to provide space to hold classes and give me free use of photocopying. It has been a win-win situation.

Shortly thereafter, the University also offered me an expense account through which I pay for travel, conferences and equipment for classes.

In October of 1996 I started with a two hour class every Tuesday night, and had six or seven regular participants for the full year, growing in number nearer the competitions as the students brought along friends and siblings. At the same time, a colleague in a nearby country asked if I would offer classes in her area. In October 1997 I continued the weekly senior classes at U.W.O. and offered weekly classes at a school in the nearby country. The class at Western continued to grow to a regular 12 to 15 students. I was also asked to help a young, and obviously talented, grade 6 student. I decided to coach him individually, and saw him weekly at his school. He has gradually developed into a possible member of the Canadian I.M.O team.

In the fall of 1998, the Tuesday senior at U.W.O continued, and due to the University and the local school board giving the class some publicity, it grew even larger. The classes in the nearby country had not flourished. After a period of alternating Mondays with that class and one at Western for grades 9 and 10 students, I changed to a cycle of two-hour, weekly classes for these students at Western. I had also had an offer from a retiring Maths Department Head, Walker Schofield, to join me, and we split the Tuesday night class between us, one hour each. Also in 1998 I started a two-hour session, one Saturday morning a month, October through April, for grades 7 and 8 students, and had a most encouraging first attendance of over 40. In the first hour I cover academic work and

Gauss contest preparation. In the second hour Mrs Margaret Kemp specialises in more hands-on work such as studies using pentominoes, tangrams, tetraflexagons and Escher type art. Mrs. Kemp is a grade four teacher who coaches and also directs the selection process for the local area Ontario Mathematics Olympian (OMO) team for grades seven and eight.

Through 1998 we expanded in numbers, and improved our communication with the mathematical community, letting more students know of our existence. U.W.O. appointed a mathematics and science liaison officer, who has assisted us in publicizing our program to all the schools within an hour's journey of the university. However, the best communication is by word of mouth. I also picked up another two students to coach individually, and arranged for two other math teachers to coach other promising students.

In 2000 we added a two hour session for grades 5 and 6 on the Saturday afternoon. As with the morning sessions I share the time, one hour each, with Mr. David McMillan, a local elementary teacher. We use the same pattern as the morning. The first hour is academic and contest preparation, and the second is hands on activities. Two other retiring Math Dept. heads, Mr. Charles Scollard and Mr. Carl Silke, have joined me in teaching the Monday sessions. I have an hour every week and they share the other hour between them.

Over the last two years, the only change in the programme is the addition of an extra hour, making three, on Tuesday nights. This enables me to concentrate on preparing students for the Olympiad level material and the American Regional Mathematics League Competition (ARML), held at Penn State in State College Pennsylvania, an eight hour drive away for us. With Peter O'Hara, a local high school teacher, my wife and I have taken students to this for the past three years. Last year we took a complete team of 15.

This school year, 2001/2002, we have a total of 128 registered in the grades 5/6 sessions, with an average attendance of 90. Of these registrants, seven are in grade 4 and one in grade 2. I think that this offers an opportunity for students to meet math peers regardless of age, and provides an atmosphere that encourages having fun with

mathematics. Many of these students have no mathematical peers in their own classrooms, some even in their own school. Forming a peer group for these talented young people is possibly the most important role of the programme. The students involved are allowed to feel that they are not the only people in their world who love mathematics. For these students' regular teachers, there is a very useful spin-off in that the books that we offer on behalf of the Waterloo Competition Group, give these students something useful and productive to do when they have finished their regular school work, which often takes the better ones a very short time. With a total registration of 76 students in the grades 7/8 sessions, we are enjoying an average of 60 per session. There are a regular 40 at the Monday sessions and 30 at the Tuesday sessions. Not all students come for every session due to other commitments. There are actually about two hundred and eighty students registered in all.

The material we use in the sessions is mainly the Canadian Mathematics Competitions from the various grades as well as the MATH Contest book from the U.S. Math League. Both organizations have kindly given us permission to copy the materials. As we study the questions in these papers we often digress into interesting areas. We also teach the students the basic concepts needed for the material. Frequently we have some of the students who are ready for concepts years before they would usually study them in school. We attempt to enable them to continue mathematically at their natural pace. Where appropriate we also have students sharing ideas they have recently learned in their own studies.

This year for the first time I have also started charging the students \$20 for the year. This money is to pay for sending them to Mathematics Competitions, as last year I had to raise a total of \$7000 to send the students to OMO and ARML. The students who went did pay some of this, as did the Provincial Mathematics Association and a very supportive local company, Trojan Technologies, which has sponsored the junior Olympiad team for several years. We also offer a snack to the younger students at the mid-time break on Saturdays.

The rewards for our efforts have been the excellent and improving results that our students have achieved on contests. Last year some students wrote contests through our Math Challenge programme, others wrote in their own schools. The ones who wrote with us scored among the

top ten teams in Canada in three of the five high school mathematics contests. We are regularly having students invited to write the higher level contests, and achieving excellent results. A personal reward has been that I have been given an honorary position in the Mathematics Department, with the title of Mathematics Co-ordinator. Another very important benefit to me is that my wife Marlene, has become progressively more interested and involved in the programme, and now takes care of all the business and attendance. This means that we work as a team, with Marlene at many of the sessions taking attendance and collecting money. For a retired couple this is a great opportunity to spend time together sharing a common interest.

For the University, the benefit is that the top mathematics students in the areas become regular visitors to the campus from a very young age. The University is also given names and addresses of the students, and they send copies of relevant information and newsletters to them personally. As a mark of their gratitude, last year the Dean hosted an appreciation dinner for the team.

I believe that a programme like this could be started in other places given the appropriate ingredients. First, you need a university which is willing to give its support to such a programme through accommodation and photocopying. Second, the university needs to help finance materials and equipment, especially for the junior students, and in publicizing the programme. You need a retired mathematics teacher or teachers who are interested in passing on their expertise and experience. I have been delighted to find that, once we got under way, the offers for assistance came in steadily, and the assistance is much appreciated. We also have three other high school and university staff who are willing to take classes for us when necessary. Peter O'Hara now also leads the ARML team, as well as assisting with covering classes when required. One benefit of this is that as a team no individual is indispensable, and as retirees we can take time off for vacations and have the others cover for us. I am hoping that when I am no longer able to run this programme there will be others willing to carry it on.

One of the most appreciated comments that we have heard is that some of the youngest students have gone straight from the class to McDonalds, where they and their friends are engrossed in mathematics, and that they

stay that way for the rest of the weekend.

I hope that this may encourage others to start a similar programme. I will be pleased to assist with advice and moral support.

I would like to take this opportunity to thank the University of Western Ontario and my colleagues for their assistance and support in this programme. Without this the programme would not be possible. I should also emphasize that all participants are unpaid volunteers.

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\* \* \*

## Ten Years of Geominatorics

*Alexander Soifer*



*Alexander Soifer is a professor at the University of Colorado. He is chair and founder of the Colorado Mathematical Olympiad, now in its eighteenth year, a member of USA Mathematics Olympiad Subcommittee, Secretary of the World Federation of National Mathematics Competitions and Editor of the Quarterly, Geombinatorics. Soifer's publications include four books, with a fifth currently in preparation.*

### 1. Conception

In the spring of 1990 I formulated some problems and conjectures in Euclidean Ramsey Theory, and I wanted to share them with a few colleagues, at least with Paul Erdős, Ron Graham and Branko Grünbaum. How would one do this? Writing several long mathematical letters, but this would take months! Sending Xerox copies, but this would seem to be so impersonal, that I do not know why would anyone read them, let alone think about them! So one day in June of 1990 I called Branko Grünbaum and proposed a solution to this dilemma:

- “What do you think about publishing a small lively journal devoted entirely to problem posing essays, work in progress, so that instead of a half dozen letters to colleagues we would write one essay?”
- “There are problem sections in many journals, but I do not know a journal completely devoted to open problems. Did you have in mind any limitations?”

- “How about geometry of combinatorial flavor, Geombinatorics?”
- “A good idea, let us try it.”
- “But I will need your essays – in every issue.”
- “ OK, I’ll try.”

Now you know “the rest of the story” of the Conception of *Geombinatorics*, which was born a year later, in June 1991, when its first issue came out. It was tiny, and contained just two essays – by Branko and myself. But the volume doubled in the second issue, when Paul Erdős and John Isbell joined the two of us. And then it took off . . .

So what is *Geombinatorics*? What sets it apart from other journals?

## 2. Live Mathematics

Academic journals remind me of old cemeteries. They publish (bury?), with a great deal of respect, completed research, solved problems. Moreover, the publications appear a couple or more years after research was completed. By then the results are of little interest, not only for the readers but even for their own authors as well, by the time the journal reached them an active mathematician would likely be working on something else! *Geombinatorics* is, perhaps, the only publication entirely dedicated to research in progress. This is a place to enjoy live mathematics!

In a sense, it is a white conspiracy of a relatively small number of active subscribers throughout the world, with an explicit goal of doing geometric combinatorics and sharing with each other their new problems, ideas, their work in progress when it still interests them – and therefore may inspire others.

Also once an issue of the journal is out, it is picked up and further disseminated to hundreds of thousands of other mathematicians by Mathematical Reviews, Zentralblatt für Mathematik, and Mathematics Abstracts.

### 3. Inclusive Accessibility

Essays in *Geombinatorics* are written to be read, by a broad spectrum of active mathematicians, from bright high school and undergraduate college students to research mathematicians. Problems we publish have classical clarity. Indeed, anyone can understand our problems – the trick is to solve them. Essays are expected to present the history of the problem, partial results, author’s conjectures and bibliography.

There is a popular belief that young students have to be kept away from starting to work on open problems. We reject this discrimination and provide young readers with a place where they can take off and sail into their own research. In fact, the doctoral thesis of Paul O’Donnell from Rutgers University grew out of a few early *Geombinatorics* publications where he wrote several essays on chromatic number of the plane, all of which appeared on the pages of *Geombinatorics*. Paul O’Donnell’s last essay [1], that was published in the year 2000, solved an old and well known problem posed in 1978 by Paul Erdős about existence of arbitrarily large girth 4-chromatic unit distance graphs. They exist!

We do have young subscribers. Moreover, we have high school and undergraduate authors. The high school student from Vermont, Thomas Pietraho, said it best (letter of August 29, 1991):

“Although I have not solved any of the problems (not yet, that is), I know that I’ll gain a lot from items such as these . . . . And if the Muses grant me their aid, I’ll be more than swift to send you my solutions.”

### 4. Subject Matter

A small publication cannot cover the breadth of today’s mathematics. Thus, *Geombinatorics* from its inception has focused on Combinatorial and Discrete Geometry and related areas. We encourage mathematicians in other areas to start similar lively publications.

We do not build fences and neither do we sit on them. *Geombinatorics* has published historical research on PJH Baudet, Van der Waerden and Schur, as well as reminiscences about Paul Erdős and Branko Grünbaum.



In some instances we are even ready to come to the rescue of folks in other creative endeavors. When the acclaimed Russian poet Evgeny Evtushenko complained that nobody was willing to publish his latest controversial poem, I offered him pages in *Geombinatorics*!

## 5. Epilogue

We have a web page:

<http://www.uccs.edu/~asoifer/geombinatorics.html>

It includes: Contents of every issue for the first 8 years, Authors Index, and the Editorial Board. In addition, prospective authors can find the Submission Rules and Copyright Form.

The success of *Geombinatorics* is due to many colleagues from all over the world who have for ten years contributed their thoughts and their aspirations to this quarterly. Some maturity manifested itself when Mathematical Reviews and Zentralblatt für Mathematik came on board and pronounced *Geombinatorics* to be their “publication of high density” (which meant that all *Geombinatorics* articles in their final form would be reviewed).

The Problem posing style of *Geombinatorics* has very much been inspired by Paul Erdős’ 20+ problem posing articles and his countless problem posing talks. Paul served as an Editor of *Geombinatorics*. Moreover, he cared about our journal: he contributed 15 essays to the 21 issues of *Geombinatorics* which appeared during his life.

Branko Grünbaum from the University of Washington has served as an Editor of *Geombinatorics* from its birth and his contributions have been critical to the success and even survival of *Geombinatorics*. Varied in topics, but uniform in highest quality and inspiration, these contributions have been the back-bone of the journal. Their regularity has been unparalleled as well: Branko has contributed 32 essays in the 37 issues which have been published to date.

*Paul Erdős in Colorado Springs*

*Branko Grünbaum, University of Washington*

A good number of essays have been inspired by Paul Erdős' and Branko Grünbaum's essays.

Our Editors Peter Johnson Jr. of Auburn University, János Pach of Courant Institute of Mathematical Sciences and Hungarian Academy of Sciences, and Jaroslav Nešetřil of Charles University of Prague, published on the pages of *Geombinatorics* wonderful mathematics themselves and brought in young and talented authors.

## Bibliography

- [1] O'Donnell, Paul, Arbitrary Girth, 4-Chromatic Unit Distance Graphs in the Plane, *Geombinatorics* **IX(3)**, 145-150, and **IX(4)**, 180-193.

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## Is it Possible a Problem Proposed in an IMO Appears in a William Lowell Putnam Competition Later?

*W Ramasinghe*

*W Ramasinghe is a Visiting Associate Professor of Ohio State University, Columbus, Ohio USA. He was the Head of Mathematics University of Colombo, Sri Lanka before coming to Ohio State on a Fulbright Scholarship. He received his PhD in Functional Analysis and an MS from The Ohio State and another MS from Ohio University. He is the founding president of the Sri Lanka National Mathematical Olympiad Association.*

This article discusses a situation that an interesting problem proposed to the 37th IMO appeared in the Sixty-first William Lowell Putnam competition in a slightly different form.

When I was leader of the Sri Lankan team, I proposed the following nice problem with the solution given to the 37th IMO held in Bombay, India.

### **Problem**

Find a sequence  $(x_n)$  of positive numbers such that

$$\sum_{n=1}^{\infty} x_n = \sum_{n=1}^{\infty} x_n^2 = 1996.$$

### **Solution**

Let  $x_n = ar^{n-1}$  with  $0 < r < 1$  for  $n \geq 1$ . Then

$$\sum_{n=1}^{\infty} x_n = \sum_{n=1}^{\infty} ar^{n-1} = \frac{a}{1-r} = 1996.$$

and

$$\sum_{n=1}^{\infty} x_n^2 = \sum_{n=1}^{\infty} a^2 (r^2)^{n-1} = \frac{a^2}{1-r^2} = 1996.$$

Solving for  $a$  and  $r$  yields  $a = 2 \cdot \frac{1996}{1997}$  and  $r = \frac{1995}{1997}$ . Notice that  $0 < r < 1$ . Hence,  $x_n = 2 \cdot \frac{1996}{1997} \left(\frac{1995}{1997}\right)^{n-1}$  for  $n \geq 1$  satisfies the requirement of the problem.

I understood very well that this problem must not have been given much attention at the 37th IMO since problems about infinite series are not of IMO type (this was my misunderstanding). However, I proposed this problem because of its appearance as an infinite series problem, it is just a simple geometric series problem. In addition, I was under the impression that students around the world learn about geometric series in grade 10 or 11 as it is the case with Sri Lankan students (though I may be wrong).

My teaching and setting of exams at the University of Colombo has been influenced by my work with the IMO. Though this problem was proposed to an IMO, I wanted to give it to undergraduates in a workable form because of the excitement it brings. Consequently, I moderated and extended the problem to a two part problem which I included in the second year undergraduate examination of the University of Colombo in the year 1997[3].

### Problem

- (i) Find  $S_n$ , in the usual notation for the series  $\sum_{k=1}^{\infty} cx^k$ , where  $c$  is a non-zero real number. Deduce the values of  $x$  for which  $\sum_{k=1}^{\infty} cx^k$  converges. Find a positive sequence  $(x_n)$  such that  $\sum_{n=1}^{\infty} x_n = \sum_{n=1}^{\infty} x_n^2 = 1997$ .
- (ii) Does there exist a positive sequence  $(x_n)$  such that  $\sum_{n=1}^{\infty} x_n = \sum_{n=1}^{\infty} x_n^2 = \frac{1}{1997}$ ? Justify your answer.

Part (i) was written in this manner to lead the undergraduates to come up with a solution similar to that of mine proposed to an IMO. The solution to part (ii) expected from the undergraduates was as follows:

There does not exist a positive sequence  $(x_n)$  such that  $\sum_{n=1}^{\infty} x_n = \sum_{n=1}^{\infty} x_n^2 = \frac{1}{1997}$ .

If  $x_n > 0$  and  $\sum_{n=1}^{\infty} x_n = \frac{1}{1997}$  then  $x_n < 1$  for  $n \geq 1$  which implies  $0 < x_n^2 < x_n$  for  $n \geq 1$ .

Therefore,  $\sum_{n=1}^{\infty} x_n^2 < \sum_{n=1}^{\infty} x_n = \frac{1}{1997}$ , and the proof is complete.

The Sixty-first William Lowell Putnam Mathematical Competition had the following problem.

### Problem

Let  $A$  be a positive real number. What are the possible values of  $\sum_{j=0}^{\infty} x_j^2$  given that  $x_0, x_1, x_2, \dots$  are positive numbers for which  $\sum_{j=0}^{\infty} x_j = A$ ?

This problem extends both my IMO and undergraduate problems, and, in fact the solution given to this problem in *Mathematics Magazine* is an extension (on the same lines) of the solution I submitted to the 37th IMO, as well as the above solution expected from undergraduates to part(ii) of the extended problem[2]. Needless to say that, regardless of the name of the competition, the flavour of the mathematics is the same in Bombay, India, Colombo, Sri Lanka and any city of the USA.

It seems that both my teaching in the university and work with IMO participants at the residential camps have helped each other without my knowledge. Isn't this one of the benefits of working for IMO? In addition to the problem solving sessions in our camps, there were some lectures with the hope of opening eyes and widening horizons of our participants about mathematics. The infinite series lecture is among those enjoyed by most of the participants.

### A Plus for Sri Lanka

The first Sri Lankan Bronze Medalist in an IMO, Missaka Warusawitharana, was also a member of the Washington University, St. Louis, USA team which received an Honorable Mention in the Sixty-first William Lowell Putnam Mathematical Competition[1].

## Acknowledgements

1. The late Professor P J O' Halloran was very generous and extremely wise to arrange for the Sri Lanka National Mathematical Olympiad Association (SLNMOA) to receive the WFNMC journal at no cost since the inception of the association. He had the wisdom to foresee that the journal would help us to educate our people, including myself, about the mathematics competitions in the world. It goes without saying that he understood the financial situation of SLNMOA. In fact, this is the only journal our association receives at the moment.
2. Many thanks also to Dr. Walter E. Mientka and the IMO Executive Committee 2001 in USA for their helping hand towards the 2001 Sri Lankan IMO team with its assistance for visa fees in our hour of need. I guess our team paid it back by winning a Bronze Medal in DC.

## References

- [1] Leonard F Klosinski, Gerald L Alexanderson, and Loren C Larson, (2001), The Sixty-first William Lowell Putnam Mathematical Competition, *The American Mathematical Monthly*, pp 841-850
- [2] The Sixty-first Annual William Lowell Putnam examination(2001), *Mathematics Magazine* 74, pp 77-83
- [3] University of Colombo Sri Lanka, Second Year Examination in Science PM 215-1996 (held in 1997)

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## WFNMC International & National Awards

### David Hilbert International Award

The David Hilbert International Award was established to recognise contributions of mathematicians who have played a significant role over a number of years in the development of mathematical challenges at the international level which have been a stimulus for mathematical learning.

Each recipient of the award is selected by the Executive and Advisory Committee of the World Federation of National Mathematics Competitions on the recommendations of the WFNMC Awards Sub-committee.

Past recipients have been: Arthur Engel (Germany), Edward Barbeau (Canada), Graham Pollard (Australia), Martin Gardner (USA), Murray Klamkin (Canada), Marcin Kuczma (Poland), Maria de Losada (Colombia), Peter O'Halloran (Australia) and Andy Liu (Canada).

### Paul Erdős National Award

The Paul Erdős National Award was established to recognise contributions of mathematicians which have played a significant role over a number of years in the development of mathematical challenges at the national level and which have been a stimulus for the enrichment of mathematics learning.

Each recipient of the award is selected by the Executive and Advisory Committee of the World Federation of National Mathematics Competitions on the recommendations of the WFNMC Awards Sub-committee.

Past recipients have been: Luis Davidson (Cuba), Nikolay Konstantinov (Russia), John Webb (South Africa), Walter Mientka (USA), Ronald Dunkley (Canada), Peter Taylor (Australia), Sanjmyatav Urjintseren (Mongolia), Qiu Zonghu (China), Jordan Tabov (Bulgaria), George Berzsenyi (USA), Tony Gardiner (UK), Derek Holton (New Zealand), Wolfgang Engel (Germany), Agnis Andžāns (Latvia), Mark Saul (USA), Francisco Bellot Rosado (Spain), János Surányi (Hungary) and Istvan Reiman (Hungary).



## **Requirements for Nominations for the David Hilbert International Award and the Paul Erdős National Award**

The following documents and additional information must be written in English:

- A one or two page statement which includes the achievements of the nominee and a description of the contribution by the candidate which reflects the objectives of the WFNMC.
- Candidate's present home and business address and telephone/telefax number.

## **Nominating Authorities**

The aspirant to the Awards may be proposed through the following authorities:

- The President of the World Federation of National Mathematics Competitions.
- Members of the World Federation of National Mathematics Competitions Executive Committee or Regional Representatives.

The Federation encourages the submission of such nominations from Directors or Presidents of Institutes and Organisations, from Chancellors or Presidents of Colleges and Universities, and others.

\* \* \*

## Tournament of Towns Corner

*Andrei Storozhev*

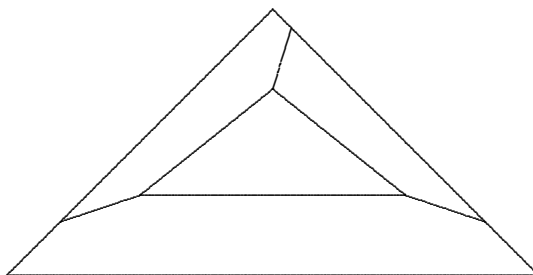
*Andrei Storozhev is a Research Officer at the Australian Mathematics Trust. He gained his PhD at Moscow State University specializing in combinatorial group theory. He is a member of the Australian Mathematics Competition Problems Committee, Australian Mathematics Olympiad Committee and one of the editors of the 'Mathematics Contests - The Australian Scene' journal.*

### Selected Problems from Tournament 23

Tournament 23 is now complete. In the second round, both Junior and Senior O Level papers consisted of five problems, and both A Level papers were made up of seven problems. Here are selected questions with solutions from this round of the Tournament.

1. Does there exist a triangle which can be dissected into four convex polygons: a triangle, a quadrilateral, a pentagon, and a hexagon?

**Solution.** The answer is yes. One such dissection is shown in the diagram below.



2. (a) There are 128 coins of two different weights, 64 of each. How can one find two coins of different weights by performing no more than 7 weighings on a balance?
- (b) There are 8 coins of two different weights, 4 of each. How can one find two coins of different weights by performing 2 weighings on a balance?

**Solution.**

- (a) Weigh 64 of the coins against the other 64. If they balance, discard one set. Weigh 32 of the remaining ones against the other 32, and continue. If they always balance, then after 6 weighings, we are down to 2 coins which must consist of a heavy one and a light one. Suppose balance is not achieved somewhere along the way. We may as well assume that it occurs at the first weighing. In the second weighing, weigh 32 coins from the heavier side against 32 coins from the lighter side. If they balance, discard these 64 coins. If not, discard the 64 coins not involved in the second weighing. Continuing this way, we will be down to 2 coins after 7 weighings. They must consist of a heavy one and a light one.
- (b) Weigh 4 of the coins against the other 4. If they balance, discard one set. Weigh 2 of the remaining 4 coins against the other 2. If they balance, take both coins from one side. If not, take 1 coin from each side. Suppose one side is heavier in the first weighing. Weigh 2 of these coins against the other 2. If they balance, all 4 are heavy. Take 1 of them and 1 from the lighter side in the first weighing. If they do not balance, then the heavier side consists of 2 heavy coins while the lighter side consists of 1 heavy and 1 light coin. We can accomplish the task by taking the 2 coins on the lighter side.
3. The sides  $AB$ ,  $BC$ ,  $CD$  and  $DA$  of a quadrilateral  $ABCD$  are tangent to a circle at the points  $K$ ,  $L$ ,  $M$  and  $N$  respectively. Let  $S$  be the point of intersection of  $KM$  and  $LN$ . Prove that if  $SKBL$  is a cyclic quadrilateral, then  $SNDM$  is also a cyclic quadrilateral.

**Solution.** Since  $BK$  and  $BL$  are tangents,  $\angle BKL = \angle KML = \angle BLK$ . Denote their common value by  $\theta$ . Then  $\angle LBK =$

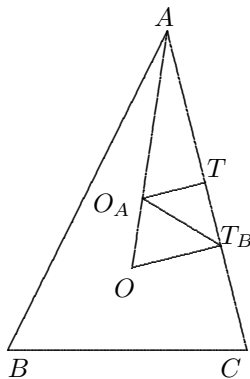


under control to a vacant square which shares a common side with its current location. The first player wins if the two white chips are located on two squares sharing a common side. Can the second player prevent the first player from winning?

**Solution.** Initially, the four chips determine a rectangle, with chips of the same colour at opposite corners. After a move by the first player from such a position, there is no victory since the two white chips are in different rows and different columns. Moreover, the four chips will no longer determine a rectangle. However, the second player can restore this position in his move. Thus there is no victory for the first player.

6. Let  $AA_1$ ,  $BB_1$  and  $CC_1$  be the altitudes of an acute triangle  $ABC$ . Let  $O_A$ ,  $O_B$  and  $O_C$  be the respective incentres of triangles  $AB_1C_1$ ,  $BA_1C_1$  and  $CA_1B_1$ . Let  $T_A$ ,  $T_B$  and  $T_C$  be the points of tangency of the incircle of  $ABC$  with sides  $BC$ ,  $CA$  and  $AB$  respectively. Prove that  $T_AO_C T_B O_A T_C O_B$  is an equilateral hexagon.

**Solution.** Since  $BCB_1C_1$  is cyclic, triangles  $ABC$  and  $AB_1C_1$  are similar. The ratio of similarity is  $\cos \alpha$  where  $\alpha = \angle CAB$ , since  $AB_1 = AB \cos \alpha$ . Let  $O$  be the incentre and  $r$  the inradius of  $ABC$ , and let  $T$  be the point of tangency of the incircle of  $AB_1C_1$  with  $AC$ .



Now  $OT_B = r$ ,  $O_A T = r \cos \alpha$ ,  $AT = AT_B \cos \alpha$ ,  $AT_B = r \cot \frac{\alpha}{2}$

and

$$TT_B = AT_B - AT = AT_B(1 - \cos \alpha) = r \cot \frac{\alpha}{2} \left( 2 \sin^2 \frac{\alpha}{2} \right) = r \sin \alpha.$$

Hence  $O_AT_B = \sqrt{O_AT^2 + T_B T^2} = r$ . By symmetry, the other sides of the hexagon are also equal to  $r$ .

7. Do there exist irrational numbers  $a$  and  $b$  such that  $a > 1$ ,  $b > 1$  and  $\lfloor a^m \rfloor = \lfloor b^n \rfloor$  for no positive integers  $m$  and  $n$ ?

**Solution.** Let  $a = \sqrt{6}$  and  $b = \sqrt{3}$ . Suppose  $\lfloor a^m \rfloor = \lfloor b^n \rfloor$  for some positive integers  $m$  and  $n$ . Denote their common value by  $k$ . Then  $k^2 \leq 6^m < k^2 + 2k + 1$  and  $k^2 \leq 3^n < k^2 + 2k + 1$ . It follows that  $2k \geq |6^m - 3^n| = 3^m |2^m - 3^{n-m}|$ . Clearly,  $n > m$  so that  $|2^m - 3^{n-m}| \geq 1$ . Hence  $2k \geq 3^m$  so that  $\frac{9^m}{4} \leq k^2 \leq 6^m$ . Now the only positive integral values of  $m$  for which  $\frac{1}{4} \leq (\frac{2}{3})^m$  holds are 1, 2 and 3. We have  $\lfloor a \rfloor = 1$ ,  $\lfloor a^2 \rfloor = 6$  and  $\lfloor a^3 \rfloor = 14$ . On the other hand,  $\lfloor b \rfloor = 1$ ,  $\lfloor b^2 \rfloor = 3$ ,  $\lfloor a^3 \rfloor = 5$ ,  $\lfloor a^4 \rfloor = 9$  and  $\lfloor a^5 \rfloor = 15$ . It follows that  $\lfloor a^m \rfloor \neq \lfloor b^n \rfloor$  for any positive integers  $m$  and  $n$ .

## World Wide Web

Information on the Tournament, how to enter it, and its rules can be obtained from the Australian Mathematics Trust web site at

<http://www.amt.canberra.edu.au/imtotgen.html>

## Books on Tournament Problems

There are four books on problems of the Tournament available. Information on how to order these books may be found on the Australian Mathematics Trust Publications pages later in this journal, or directly via the Trust's web site.

Please note the Tournament's postal address in Moscow:

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\* \* \*

## For Your Bookshelf

### *The Geometry of Numbers*

C.D.Olds, Anneli Lax and Giuliana Davidoff  
The Mathematical Association of America 2000  
176 + xii pp. paperbound

Imagine a square lattice in the plane. Now ask yourself which lines would have lattice points on them and how many. Ask which strips would zap through the plane without touching a lattice point. What would be the largest area of a rectangle without lattice points inside it? How is the area of a polygon with vertices in lattice points tied with the number of lattice points on its boundary and interior?

These questions are intriguing. Their solutions are exciting. But besides all this, exposition is a delight. Every solution results in new questions, which in turn, once solved, provide insight for next generation of questions. This “train-of-thought approach” throughout the book, makes for a most enjoyable active reading-solving experience.

The authors prove a number of non-trivial results, and appropriately refer to outside sources for proofs which would have taken them too far from their main subject or into too deep a waters of mathematical sea.

In part II the book reaches a higher level of sophistication. It presents Minkowsky and Blichfeldt theorems, and their applications to approximations and evaluation of quadratic forms.

The history of creation of this book would make for an exciting story too. Started by the late C. D. Olds (1912-1979), it continued by Anneli Lax, who passed away in 1999, and was finished by Giuliana Davidoff in 2000. And the Gaussian Integers Appendix was written by Peter D. Lax.

This book would be a welcome addition for all active mathematicians, from high school to college, and from students to professors but do not forget to keep your pencil and paper handy while reading these pages!



Competition people would find here a rich material for problem solving and problem posing as well.

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