A Retrospective View of the 1989 IMO

P.J. O'Halloran

P.J. O'Halloran was Chairman of the 1988 International Mathematical Olympiad Organising Committee. He is President of the World Federation of National Mathematics Competitions, Executive Director of the Australian Mathematics Competition and Chairman of the Asian Pacific Mathematics Olympiad

1. Introductory Comments

The 30th International Mathematical Olympiad was successfully held in Braunschweig, Federal Republic of Germany, from 13 - 24 July, 1989. There was a record number of 291 contestants from 50 countries plus representatives from Japan and Denmark.

The IMO, the pinnacle of excellence and achievement for school students of mathematics throughout the world, includes amongst its goals:

* to discover, encourage and challenge mathematically gifted school students;
* to foster friendly international relations between students, teachers and mathematicians; and
* to share information on educational systems, mathematical curricula and pedagogy.

The 1989 IMO certainly helped to fulfil these goals. One of the highlights of the Olympiad for the author was the inspiring speech that the 1989 IMO Chairman, Professor Arthur Engel, gave at the closing Ceremony (see 5. in this article).

One hundred and forty-seven contestants from 43 of the 50 participating countries achieved medals (20 gold, 55 silver and 72 bronze ... see Table Three). While another 66 received honourable mention awards by obtaining a perfect score of 7 for at least one of the IMO questions. This meant that 48 teams had students who had received some kind of merit award.

It was generally considered (and was confirmed by the statistics) that this year's questions were somewhat easier than last year with an overall mean score per question of 3·1 compared with 2·5 in 1988. (see Tables One and Four). At the same time, the questions were generally better overall discriminators (see Table Two).

There is always discussion at IMOs about criteria for student eligibility at future Olympiads. One such criterion is the eligibility or not of students who have received gold medals at previous Olympiads. Table Four was produced in order to obtain some measure of this occurrence. It is the author's view that once students have obtained the ultimate, a gold medal, they should give other students the opportunity to represent their country.

2. A Selection of 30th IMO Tables of Statistics

The following tables were mainly generated by the software computer package designed for the 1988 IMO. They are only a selection of the statistics that can be generated as long as the appropriate data is available.

TABLE ONE:

1989 IMO Score Frequency Per Question (291 Candidates)

Marks	**Number of students per Mark per Question**					
	Q1	**Q2**	**Q3**	**Q4**	**Q5**	**Q6**
0	109	45	188	65	122	143
1	31	23	27	37	3	23
2	29	9	19	12	3	13
3	4	49	13	23	3	6
4	4	14	2	8	2	11
5	8	28	7	9	8	15
6	15	15	3	5	8	11
7	91	108	32	132	142	69
Mean	3.0	4.2	1.3	4.0	3.8	2.5

Table One displays the spread of score frequencies obtained by the contestants for each question. The statistics clearly show the level of difficulty for each question. By far the most difficult question in this year's IMO was Question Three though this was in no way as difficult as Question Six in the 1988 IMO.

TABLE TWO:

Discrimination Factors of 1989 IMO Questions

Group of Students	**Question**						**# in Group**
	Q1	**Q2**	**Q3**	**Q4**	**Q5**	**Q6**	
Gold Medallists	0.00	0.00	0.20	0.00	0.00	0.00	20
Gold and Silver	0.39	0.11	0.83	0.11	0.06	0.22	75
All Medallists	0.53	0.31	0.67	0.44	0.25	0.58	147
All Contestants	0.83	0.93	0.50	0.89	0.96	0.86	291

Table Two gives the biserial discrimination index for each question. To obtain this index, the contestants are first ranked in overall ability. Then for a particular question, the number of marks greater than or equal to 4 for the top 25% of contestants is compared with the bottom 25% of contestants. The index can take values from -1 or +1. A positive value indicates that more contestants in the top group obtained 4 or more marks than contestants in the bottom group. It is considered that questions with discriminating factors of at least 0.4 have discriminated effectively.

In particular, Question Five was the best overall discriminator. However Question Three had the lowest mean score, and the lowest discriminator factor overall. At the same time it was, by far, the best discriminator for the combined gold and silver medallist group. Unlike Question Six in the 1988 IMO, there was no significant discriminating question among the gold medallist group in the 1989 IMO.

TABLE THREE:

1989 IMO Distribution of Awards

Country	Team Size	Code	Score	Medals Gold	Silver	Bronze	Honourable Mention
Australia	6	AUS	119		2	2	
Austria	6	AUT	111		2	1	1
Belgium	6	BEL	104			3	2
Brazil	6	BRA	64			3	
Bulgaria	6	BUL	195	1	3	2	
Canada	6	CAN	123		1	3	2
Colombia	6	COL	119		1	2	3
Cuba	6	CUB	69			1	3
Cyprus	6	CYP	24				1
Czechoslovakia	6	CZE	202	2	1	3	
Finland	6	FIN	58				3
France	6	FRA	156		1	5	
Federal Republic Of Germany	6	FRG	187	1	3	2	
German Democratic Republic	6	GDR	216	3	2	1	
Greece	6	GRE	122		1	3	2
Hong Kong	6	HKG	127		2	1	1
Hungary	6	HUN	175		4	1	1
Iceland	4	ICE	33				2
Indonesia	6	INA	21				
India	6	IND	107			4	1
Iran	6	IRA	147		2	3	1
Ireland	6	IRE	37				2
Israel	6	ISA	105		2	1	
Italy	6	ITA	124		1	2	3
Kuwait	6	KUW	31				
Luxemburg	3	LUX	65		1	1	
Mexico	6	MEX	79			1	3
Morocco	6	MOR	63			1	3
Netherlands	6	NET	92		1	1	2
Norway	4	NOR	64			1	2

TABLE THREE (Cont'd)

1989 IMO Distribution of Awards

Country	Team Size	Code	Score	Medals Gold	Silver	Bronze	Honourable Mention
New Zealand	6	NZL	69			2	2
Peru	6	PER	51				3
Philippines	6	PHI	45		1		
Poland	6	POL	157		3	3	
Portugal	6	POR	39				4
Peoples Republic Of China	6	PRC	237	4	2		
Republic Of Korea	6	ROK	97		1		4
Romania	6	ROM	223	2	4		
Singapore	6	SIN	143			4	2
Spain	6	SPA	61			1	4
Sweden	6	SWE	73			2	1
Thailand	6	THA	54			1	2
Tunesia	6	TUN	81		1		2
Turkey	6	TUR	133		1	4	1
United Kingdom	6	UNK	122		2	1	2
United States Of America	6	USA	207	1	4	1	
Union Of Soviet Socialist Repu	6	USS	217	3	2	1	
Venezuela	4	VEN	6				
Vietnam	6	VIE	183	2	1	3	
Yugoslavia	6	YUG	170	1	3	1	1

Table Three summarises the teams merit award achievements at the 1989 IMO recording the number of gold, silver and bronze medallists as well as the number of honourable mention awards for each team.

TABLE FOUR:

Comparison of the 1981-1989 Results

Olympiad	22nd	23rd	24th	25th	26th	27th	28th	29th	30th
Year	1981	1982	1983	1984	1985	1986	1987	1988	1989
Host Country	USA	Hungary	France	Czech.	Finland	Poland	Cuba	Australia	FRG
Team Size	8	4	6	6	6	6	6	6	6
# Countries	27	30	32	34	38	37	42	49	50
# Candidates	185	119	186	192	209	210	237	268	291
Total # Medallists	56% 103	51% 61	50% 93	51% 98	48% 101	51% 107	51% 120	45% 120	50% 147
# Gold Medallists *(range of scores)*	19% 36 *(42-41)*	8% 10 *(42-37)*	5% 9 *(42-37)*	7% 14 *(42-40)*	7% 14 *(42-34)*	9% 18 *(42-34)*	9% 22 *(42-42)*	6% 17 *(42-32)*	7% 20 *(42-38)*
# Silver Medallists *(range of scores)*	20% 37 *(40-34)*	17% 20 *(36-30)*	15% 27 *(36-26)*	18% 35 *(39-26)*	17% 35 *(33-22)*	20% 41 *(33-26)*	18% 42 *(41-32)*	14% 38 *(31-23)*	19% 55 *(30-37)*
# Bronze Medallists *(range of scores)*	16% 30 *(33-27)*	26% 31 *(29-22)*	31% 57 *(25-15)*	26% 49 *(25-17)*	25% 52 *(21-15)*	23% 48 *(25-17)*	24% 56 *(31-18)*	24% 65 *(22-14)*	25% 72 *(18-29)*
# Special Prizes	?	?	?	?	0	1	0	1	0
# Honourable Mentions	-	-	-	-	-	-	-	33	65
Average Score	26.6	20.8	15.3	18.7	14.9	18.1	19.9	15.0	18.8

TABLE FIVE:

Achievements of 1989 IMO Contestants at Previous IMOs

At the 1989 International Mathematical Olympiad there were 291 contestants.
Of these:
75 participated at the 1988 IMO,
13 at the 1987 IMO,
4 at the 1986 IMO and
1 at the 1985 IMO.

1988	1989				
	G	S	B	HM	—
5 Gold	4	1			
15 Silver	6	8	1		
25 Bronze	1	9	7	7	1
9 H.M.	1	1	3	4	
21 —		3	4	7	7
75 Contestants	12	22	15	18	8

1987	1988					1989				
	G	S	B	HM	—	G	S	B	HM	—
5 Gold	1	4				5				
3 Silver		2	1			2	1			
2 Bronze		1	1				1		1	
3 —			3					1	2	
13 Contestants	1	7	5	0	0	7	2	1	3	0

1986	1987					1988					1989				
	G	S	B	—	NP	G	S	B	HM	—	G	S	B	HM	—
1 Gold	1						1				1				
1 Silver					1	1						1			
0 Bronze															
2 —					2			2					1	1	
4 Contestants	1	0	0	0	3	1	1	2	0	0	1	1	1	1	0

There was only one 1989 contestant who participated in 4 previous Olympiads. His results were as follows:-

1985 —— Bronze
1986 —— Gold
1987 —— Gold
1988 —— Silver
1989 —— Gold

The statistics in **Table Five** show that 75 (or 26%) of the 1989 IMO contestants had participated at the 1988 IMO and of these, 12 received gold, 22 silver and 15 bronze medals with an additional 18 achieving honourable mentions.

TABLE SIX:

The IMO - A Truly International Event

Year	Europe	Asia	North America	South Amercia	Africa	Oceania	TOTAL
1959	7						7
1960	5						5
1961	6						6
1962	7						7
1963	8						8
1964	8	1					9
1965	9	1					10
1966	8	1					9
1967	12	1					13
1968	11	1					12
1969	13	1					14
1970	13	1					14
1971	13	1	1				15
1972	12	1	1				14
1973	14	1	1				16
1974	14	2	2				18
1975	14	2	1				17
1976	15	1	2				18
1977	17	1	2		1		20
1978	12	3	2				17
1979	18	2	2	1			23
1980	There was no Olympiad in 1980						-
1981	17	1	4	3	1	1	27
1982	17	5	3	2	2	1	30
1983	20	3	3	2	3	1	32
1984	22	3	3	2	3	1	34
1985	22	7	3	2	3	1	38
1986	22	6	3	2	3	1	37
1987	23	6	6	4	2	1	42
1988	24	11	4	5	3	2	49
1989	25	13	4	4	2	2	50

The dominance of European participation in the early years of the IMO is clearly shown. With greater international involvement this dominance has been reduced until in 1988, for the first time in the history of the IMO, more than 50% of the competing countries were non-European. The participation in the 1989 IMO remained at approximately the same level. The IMO has truly become a world-wide international annual event.

TABLE SEVEN:

IMO Participation of Countries

EUROPE	Y	P	H
Romania	1959	30	4
Hungary	1959	29	3
Czechoslovakia	1959	30	3
Bulgaria	1959	30	2
Poland	1959	29	3
USSR	1959	27	3
German D R	1959	28	2
Yugoslavia	1963	26	2
Finland	1965	16	1
Great Britain	1967	22	1
Sweden	1967	22	-
Italy	1967	10	-
France	1967	20	1
Belguim	1969	12	-
Netherlands	1969	19	-
Austria	1970	19	1
Greece	1975	12	-
F R Germany	1977	12	1
Luxembourg	1979	8	-
Spain	1983	7	-
Cyprus	1984	6	-
Norway	1984	6	-
Iceland	1985	5	-
Ireland	1988	2	-
Portugal	1989	1	-

ASIA	Y	P	H
Mongolia	1964	19	-
Vietnam	1974	13	-
Turkey	1978	6	-
Israel	1979	8	-
Kuwait	1982	8	-
Iran	1985	5	-
China	1985	5	-
Hong Kong	1988	2	-
Indonesia	1988	2	-
Philippines	1988	2	-
Korea	1988	2	-
Singapore	1988	2	-
India	1989	1	-
Thailand	1989	1	-

S. AMERICA	Y	P	H
Brazil	1979	10	-
Colombia	1981	9	-
Venezuela	1981	3	-
Uruguay	1987	1	-
Peru	1987	3	-
Argentina	1988	1	-
Ecuador	1988	1	-

N. AMERICA	Y	P	H
Cuba	1971	17	1
USA	1974	15	1
Mexico	1981	4	-
Canada	1981	9	-
Panama	1987	1	-
Nicaragua	1987	1	-

OCEANIA	Y	P	H
Australia	1981	9	1
New Zealand	1988	2	-

AFRICA	Y	P	H
Algeria	1977	8	-
Tunisia	1981	8	-
Morocco	1983	7	-

This Table shows for each IMO participating country, its first year at an Olympiad (**Y**), the number of times it has participated (**P**) and the number of times it has hosted an IMO (**H**).

The Opening Ceremony of the 30th IMO in Braunschweig FRG

TABLE EIGHT:

1989 IMO Countries Rankings By Mean Scores Per Question

COUNTRY	Q1	Q2	Q3	Q4	Q5	Q6	Overall
Australia*	29	12	26	32	19	18	23
Austria	18	25	41	29	33	13	25
Belgium	16	28	32	28	25	27	28
Brazil	25	34	27	37	39	42	37
Bulgaria	25	3	5	1	1	5	7
Canada	10	34	29	26	12	25	20
Colombia	23	27	29	16	19	22	23
Cuba	45	30	32	41	33	29	35
Cyprus	47	41	32	48	44	45	48
Czechoslovakia	5	10	10	10	1	1	6
Finland	43	46	41	23	35	45	40
France	9	10	11	25	9	31	13
Federal Republic of Germany	3	15	9	7	1	11	8
German Democratic Republic	3	5	2	7	1	7	4
Greece	39	44	15	18	23	22	21
Hong Kong	23	17	24	23	27	13	18
Hungary	8	6	13	14	11	16	10
Iceland (4)	34	34	32	29	44	45	43
Indonesia	44	47	41	46	44	42	49
India	25	19	20	34	19	35	26
Iran	11	12	17	18	18	17	14
Ireland	45	49	27	34	44	31	46
Israel	30	30	13	41	27	11	27
Italy	20	17	37	14	24	20	19
Kuwait	48	43	15	45	44	45	47
Luxembourg (3)	16	19	5	18	19	38	17
Mexico	39	34	20	39	33	25	33
Morocco	48	25	41	44	27	37	38
Netherlands	28	39	19	18	27	38	31
Norway (4)	11	44	41	37	17	21	30
New Zealand	39	42	20	29	36	34	35
Peru	39	38	37	29	44	38	42
Philippines	33	45	41	43	40	36	44
Poland	20	19	17	13	12	10	12
Portugal	20	49	37	40	43	45	45
Peoples Republic of China	1	1	1	3	1	3	1
Republic of Korea	34	22	41	16	42	13	29
Romania	2	1	3	3	1	3	2
Singapore	15	19	20	10	16	22	15
Spain	38	22	29	46	40	31	39
Sweden	18	39	37	50	27	27	34
Thailand	48	34	41	27	37	42	41
Tunisia	37	16	32	34	38	29	32
Turkey	34	12	41	7	25	8	16
United Kingdom	30	32	24	18	15	16	21
United States of America	8	8	11	1	1	2	5
Union of Soviet Socialist Republics	8	3	3	3	9	7	3
Venezuela (4)	50	48	41	49	44	38	50
Peoples Republic of Vietnam	11	6	7	3	1	18	9
Yugoslavia	11	22	7	10	12	9	11

* Unless specified, the number of Contestants in a team is 6.

Table Eight summarises the rankings based on the mean scores per question for each country on the six questions. It could be used as an indication of the strengths of the teams in the various mathematics topics which were tested at the 1989 IMO. The overall mean rankings for the countries are different from the basic rankings for some countries due to some teams not having six students.

3. The 1989 IMO Question Papers

XXX. INTERNATIONALE
MATHEMATIK-OLYMPIADE

13.–24. Juli 1989

Bundesrepublik Deutschland
Braunschweig · Niedersachsen

INTERNATIONALE MATHEMATIK OLYMPIADE

English version

FIRST DAY

Braunschweig, July 18th 1989

1. Prove that the set $\{1, 2, \ldots, 1989\}$ can be expressed as the disjoint union of subsets A_i ($i = 1, 2, \ldots, 117$) such that
   (i) each A_i contains 17 elements;
   (ii) the sum of all the elements in each A_i is the same.

2. In an acute-angled triangle ABC the internal bisector of angle A meets the circumcircle of the triangle again at A_1. Points B_1 and C_1 are defined similarly. Let A_0 be the point of intersection of the line AA_1 with the external bisectors of angles B and C. Points B_0 and C_0 are defined similarly.
   Prove that
   (i) the area of the triangle $A_0B_0C_0$ is twice the area of the hexagon $AC_1BA_1CB_1$;
   (ii) the area of the triangle $A_0B_0C_0$ is at least four times the area of the triangle ABC.

3. Let n and k be positive integers and let S be a set of n points in the plane such that
   (i) no three points of S are collinear, and
   (ii) for every point P of S there are at least k points of S equidistant from P.
   Prove that
   $$k < 1/2 + \sqrt{2n}.$$

Time: 4.5 hours
Each problem is worth 7 points.

XXX. INTERNATIONALE MATHEMATIK-OLYMPIADE

13.–24. Juli 1989

Bundesrepublik Deutschland
Braunschweig · Niedersachsen

English version

SECOND DAY

Braunschweig, July 19th 1989

4. Let ABCD be a convex quadrilateral such that the sides AB, AD, BC satisfy $AB = AD + BC$.
There exists a point P inside the quadrilateral at a distance h from the line CD such that $AP = h + AD$ and $BP = h + BC$.
Show that

$$\frac{1}{\sqrt{h}} \geqq \frac{1}{\sqrt{AD}} + \frac{1}{\sqrt{BC}}.$$

5. Prove that for each positive integer n there exist n consecutive positive integers none of which is an integral power of a prime number.

6. A permutation $(x_1, x_2, \dots, x_{2n})$ of the set $\{1, 2, \dots, 2n\}$, where n is a positive integer, is said to have property P if $|x_i - x_{i+1}| = n$ for at least one i in $\{1, 2, \dots, 2n-1\}$.
Show that, for each n, there are more permutations with property P than without.

Time: 4.5 hours
Each problem is worth 7 points.

4. Notes on the Origin of Some of the IMO Questions

QUESTION 1:

This question is based on the paper, "Disjoint Subsets having a Constant Sum" jointly written by Professors Kujoshi Ando (Nippon Medical School, Kawasaki, Japan), Severino Gervacio (Iligan Institute of Technology, Philippines) and Mikio Kano (Akashi Technological College, Hyogo, Japan). The paper will appear in a future issue of the "Discrete Mathematics " Journal. The question was modified by the Filipino co-author of the paper.

QUESTION 2:

This is the second question proposed by an Australian which has been included in an IMO. It was proposed by Mrs Esther Szekeres (Macquarie University, Sydney, Australia). The previous question was proposed by her husband Professor George Szekeres at the 1985 IMO in Finland.

Regarding the source of the question Mrs Esther Szekeres stated, "The problem proposed was the outcome of several years of geometric problem-setting. I have the chance and inspiration to do this in the enrichment out-of-school classes that a number of my colleagues are involved with on a weekly basis here in Sydney. The students are an enthusiastic but inhomogenous group ranging from 14 to 17 years old. Naturally the questions discussed must be approachable to the average good student in the group.

In the course of this and other similar activities, I inevitably find problems that are too difficult to set for the relevant group. So I set these problems aside for a more suitable opportunity. This particular problem seemed to me to be the correct order of difficulty for the Olympiads and so I suggested the problem to the Jury."

QUESTION 3:

This question was proposed by Harm Derksen, a first year undergraduate student at the University of Nijmegen, Netherlands. He was a Bronze Medallist at the 1988 IMO in Australia.

QUESTION 4:

The proposer of this question was Professor Eggert Briem (University of Iceland, Reykj, Iceland).

QUESTION 5:

Professor Berndt Lindstrom (Royal Institute of Technology, Stockholm, Sweden) was the proposer of this question. Professor Lindstrom said the idea for the problem originated from studying the irregularities of the distribution of primes.

[Question 5 is not connected with his present research. He is referring to Ernst Trots: Primzahlen, Verlag Birkhauser, 1953 page 40.]

QUESTION 6:

The proposer of this question was Professor Marcin Kuczma (Institute of Mathematics, University of Warsaw, Poland) who was the Deputy Leader of the 1989 Polish IMO Team (Incidentally he is also the Editor of the excellent Polish journal for students "Delta").

The XXX IMO is over. But long before that preparations for the next IMO have started. As usual, at the end of the current olympiad, the host for the next olympiad presents its oral invitation. The IMO has a Code of Honour: A host country *must* invite at least all countries that have ever participated in the IMO. Even if it has no diplomatic relations to an IMO-participant. Countries with official observers at the preceding olympiad should also be invited.

We were especially keen on Japanese participation. We succeeded to get observers. Japanese competition is usually strong competition, and competition is healthy for all of us. It will speed up the development of an open IMO.

6. Conclusion

Following Professor Engel's strong plea for exchange of Olympiad information, the Federation intends to obtain relevant information from all Team Leaders of the 1989 IMO participating countries, and publish a special book later next year. (A special request brochure accompanies this issue).

In conclusion it should again be emphasised that besides the discovering, encouraging and challenging of mathematically gifted school students, the olympiads aim at the fostering of friendly international relations and at the exchanging of ideas between students, teachers and mathematicians. There is no doubt that these aims were achieved at the 1989 IMO in Braunschweig in the Federal Republic of Germany. Congratulations to all the organisers and participants.

Professor Engel presenting a bronze medal to Brian Weatherson (Australia)